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REMARKS ON THE MODEL OF WEINBERG II

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Contents

We re-investigate the model of Weinberg, and we work out some features of this model, which can be used to construct other collapsing models.

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1. INTRODUCTION

In this paper we will deepen the geometric background of a collapsing star and we will investigate its behavior at the event horizon in more detail. Special attention is given to the construction of the stress-energy-momentum tensor in the non-comoving system.

The model of Weinberg describes a non-rotating star consisting of pressure-free incoherent matter collapsing in free fall. For the understanding of the model it is useful to apply geometrical methods. Since the star is to be surrounded by the static Schwarzschild field, we interpret the exterior space-like part of the model as Flamm's paraboloid and the interior geometry as a cap of a sphere which is attached to a suitable position on Flamm's paraboloid. During the collapse of the star the cap of the sphere slides down Flamm's paraboloid, while the exterior Schwarzschild field remains unchanged according to Birkhoff’s theorem. Based on this geometric model some relations which can be derived from the collapsing metric become obvious. They can be decomposed into components which can be attributed to the cap of the sphere or to Flamm's paraboloid, respectively. In addition, it will be explained by this view why the surface of the stellar object cannot go below the Schwarzschild event horizon. Certain limitations are associated to the scale factor, commonly used in literature, which we will discuss later on.

2. THE GEOMETRIC FOUNDATIONS

The line element of Weinberg, can be written in the form

$$ds^2 = \mathcal{K}^2 \left[\frac{1}{1-\frac{r'^2}{\mathcal{R}_0^2}} dr'^2 + r'^2 d\vartheta^2 + r'^2 \sin^2\vartheta \, d\varphi^2 \right] - dt'^2 \ . \tag{2.1}$$

Therein $\mathcal{K} = \mathcal{K}(t')$ is the time-dependent dimensionless scale factor, which describes the collapse, $\{r', t'\}$ are the comoving co-ordinates. For the radial non-comoving co-ordinate

$$r = \mathcal{K} r' \tag{2.2}$$

applies. The relation of the time t' to the non-comoving co-ordinates, however, has proved to be problematic. From the metric we read the 4-bein

$$\overset{1'}{e}_{1'} = \mathcal{K}\alpha_I, \quad \overset{2'}{e}_{2'} = \mathcal{K} r', \quad \overset{3'}{e}_{3'} = \mathcal{K} r' \sin\vartheta, \quad \overset{4'}{e}_{4'} = 1, \quad \alpha_I = \frac{1}{\sqrt{1-\frac{r'^2}{\mathcal{R}_0^2}}}, \quad a_I = \frac{1}{\alpha_I} \ . \tag{2.3}$$

$\mathcal{R}_0 = \text{const.}$ is the radius of the cap of a sphere at the time $t'=0$, hence at the beginning of the collapse. The reciprocal 4-bein is easy to calculate due to the diagonality of the metric. Usually the force of gravity of a gravitational system is derived from the time-like metric factor

$$'A_{4'1'}{}^{4'} = -\overset{4'}{e}_{4'} e^{4'}_{4'|1'} = -'E_{1'} .$$

Since the metric factor of the Weinberg-line element is $\overset{4'}{e}_{4'}=1$, there are no perceptible acceleration forces in the comoving system. It is $'E_{1'}=0$. The collapse takes place in free fall.

If we temporarily omit the primes which mark the comoving system the stress-energy-momentum tensor has the simple form

$$T_{mn} = -p g_{mn} + (p + \mu_0) u_m u_n , \qquad (2.4)$$

where is p the pressure, μ_0 the mass density, and u_m the velocity of the particles. From the conservation law

$$T^{\alpha m}{}_{||m} = T^{\alpha m}{}_{|m} + A_{mn}{}^{\alpha} T^{nm} + A_m T^{\alpha m} = -g^{\alpha\beta} p_{|\beta} - A_{\beta\gamma}{}^{\alpha} p g^{\beta\gamma} + A_{44}{}^{\alpha}\mu_0 - A_\beta g^{\alpha\beta} p = 0, \quad \alpha = 1,2,3$$

results with $-A_{\beta\gamma}{}^{\alpha} g^{\beta\gamma} = A^{\alpha}$ and $A_{44}{}^{\alpha} = 0$ for the pressure the condition $p_{|\alpha} = 0$. Since the pressure in a collapsing object cannot be constant, the model must be pressure-less. Again, this is only possible for non-coherent dust, but only as long as the particles do not move too close during the collapse. The pressure-free state is a direct consequence of the ansatz $\overset{4'}{e}_{4'}=1$. The fact that the co-ordinate time t' coincides with the proper time T' of the observer is only valid for an observer which comes in free fall from *infinity*. This is evidently not the case for the surface of the collapsing star. The use of the proper time of an observer who does not participate in the actual collapse ultimately leads to the violation of the addition theorem of velocities and to the destruction of the Lorentz relations, as we will discuss later on.

Although the model of Weinberg already contains inconsistencies concerning its ansatz, we want to further explore the model, because it gives rise to several interesting mechanisms which can be stimulating in the construction of other models. Furthermore, when the surface of the object has reached the event horizon, the very problems known from the Schwarzschild theory occur. The collapse velocity reaches the speed of light at this location.

The metric of the non-comoving system gives more insight into this problem

$$\text{(B)} \quad ds^2 = \alpha^2 dr^2 + r^2 d\vartheta^2 + r^2 \sin^2\vartheta\, d\varphi^2 + a_T^2 dit^2 . \qquad (2.5)$$

From this we read the 4-bein

$$\overset{1}{e}_1 = \alpha_R , \quad \overset{2}{e}_2 = r, \quad \overset{3}{e}_3 = r\sin\vartheta, \quad \overset{4}{e}_4 = a_T, \quad \alpha_R = \frac{1}{a_R} = \frac{1}{\sqrt{1-\frac{r^2}{\mathcal{R}_g^2}}} . \qquad (2.6)$$

We have little reliance on the time-like metric factor a_T which Weinberg has noted in his textbook. $\mathcal{R}_g$ is the radius of the cap of a sphere and is at the time $t' = \text{const.}$ at all points of the cap by definition equally large. It is closely linked with the radius of curvature of the Schwarzschild parabola on the boundary surface to Flamm's paraboloid

$$\rho_g = \sqrt{\frac{2r_g^3}{M}} \,. \tag{2.7}$$

If ρ_g is extended to the directrix of the Schwarzschild parabola, then $\mathcal{R}_g$ is cut out on this straight line. In addition, we get from the properties of the Schwarzschild parabola the simple relation

$$\rho_g = 2\mathcal{R}_g, \quad \mathcal{R}_g = \sqrt{\frac{r_g^3}{2M}} \,, \tag{2.8}$$

wherewith we have determined the radius of curvature of that cap which is matched to the Schwarzschild parabola. r_g is the radial non-comoving co-ordinate on the boundary surface. It changes its value during the collapse.

With the polar angle η and

$$r = \mathcal{R}_g \sin\eta \tag{2.9}$$

it is possible to bring the metric (B) into the form

$$(B') \quad ds^2 = \mathcal{R}_g^2 d\eta^2 + \mathcal{R}_g^2 \sin^2\eta d\vartheta^2 + \mathcal{R}_g^2 \sin^2\eta \sin^2\vartheta d\varphi^2 - a_T^2 dt^2 \,. \tag{2.10}$$

In [2] we have derived the relation

$$\mathcal{R}_g = \sqrt{\mathcal{K}^3}\,\mathcal{R}_0, \quad r' = \mathcal{R}_0 \sin\eta' \,. \tag{2.11}$$

η' is the comoving polar angle. Thus, we also have for the metric (A)

$$ds^2 = \mathcal{K}^2\left[\mathcal{R}_0^2 d\eta'^2 + \mathcal{R}_0^2 \sin^2\eta' d\vartheta^2 + \mathcal{R}_0^2 \sin^2\eta' \sin^2\vartheta d\varphi^2\right] - dt'^2 \tag{2.12}$$

and with $dt' = \mathcal{K}\mathcal{R}_0 d\psi$ a simple form for the line element $ds = \mathcal{K}(t') ds_0$, where ds_0 is the line element at the time $t' = 0$.

For the understanding of the model, it is important to calculate the collapse velocity. From (2.2) one gets

$$dr = \mathcal{K} dr' + \mathcal{K}^{\bullet} r' dt', \quad \mathcal{K}^{\bullet} = \frac{\partial \mathcal{K}}{\partial t'} \,. \tag{2.13}$$

Thus, we can calculate components

$$\Lambda^1_{1'} = \mathcal{K}, \quad \Lambda^1_{4'} = i\frac{r'}{\mathcal{R}_0}\sqrt{\frac{1}{\mathcal{K}} - 1} \tag{2.14}$$

from the coefficients of the co-ordinate transformation $\Lambda^i_{i'} = x^i_{|i'}$ connecting the comoving and non-comoving co-ordinate systems. This is possible because one knows from the solution of Einstein's field equations the relation

$$\mathcal{K}^{\cdot} = -\frac{1}{\mathcal{R}_0}\sqrt{\frac{1}{\mathcal{K}} - 1}\ . \qquad (2.15)$$

Transvecting the Λ with the 4-bein of the metric (A) or (B), respectively

$$L^{m}_{m'} = \overset{m}{e}_{i}\, \Lambda^{i}_{i'}\, \underset{m'}{e}^{i'} \qquad (2.16)$$

one obtains a matrix which transforms the 4-bein of the two systems. It is the Lorentz transformation with the coefficients

$$L^{1}_{1'} = \alpha_C, \quad L^{1}_{4'} = -i\alpha_C v_C, \quad L^{4}_{1'} = i\alpha_C v_C, \quad L^{4}_{4'} = \alpha_C, \qquad (2.17)$$

wherein is

$$\alpha_C = \frac{\alpha_R}{\alpha_I}, \quad v_C = \alpha_I v_I \sqrt{\frac{1}{\mathcal{K}} - 1}, \quad v_I = -\frac{r'}{\mathcal{R}_0}. \qquad (2.18)$$

If one puts v_I under the root and if one uses the relations (2.2) one has recognized with

$$\alpha_I = \frac{1}{\sqrt{1 - v_I^2}}, \quad v_C = \alpha_I \sqrt{v_R^2 - v_I^2}, \quad v_R = -\frac{r}{\mathcal{R}_g} \qquad (2.19)$$

that v_C is composed of two velocities, but is violating Einstein's addition theorem of velocities. The latter would read as

$$\alpha_I = \alpha_R \alpha_I \left(1 - v_R v_I\right), \quad v_C = \frac{v_R - v_I}{1 - v_R v_I}\ . \qquad (2.20)$$

If the surface of the collapsing star has reached the Schwarzschild event horizon $r = 2M$, it follows from (2.8) that $\mathcal{R}_g = 2M$. The cap of the sphere is now a hemisphere and joins with its edge the circle at the waist of Flamm's paraboloid. If one allowed a further contraction, the cap of the sphere would unsolder from Flamm's paraboloid, the linking condition would not be satisfied any longer, the geometric picture would be destroyed.

On the boundary surface the Lorentz factor α_R is equal to

$$\alpha_R^g = \frac{1}{\sqrt{1 - \frac{2M}{r_g}}}$$

and is imaginary for $r_g < 2M$ and also α_C of (2.18). In addition, it can be seen from (2.19) with (2.8) that the velocity of the boundary surface at $2M$ reaches the velocity of light. Below the event horizon the star would collapse faster than light, gravity would be imaginary.

We will avoid such considerations which are often made in the context of the Schwarzschild theory, and we will limit the range of validity of the model in such a way that the model remains in the causal region. However, such a forced restriction reduces the plausibility of the model.

However, this limitation prevents the star from shrinking to a point singularity with an infinitely high mass density and infinitely high spatial curvature. Although this is a concept which many physicists admit, we are not prepared to join it.

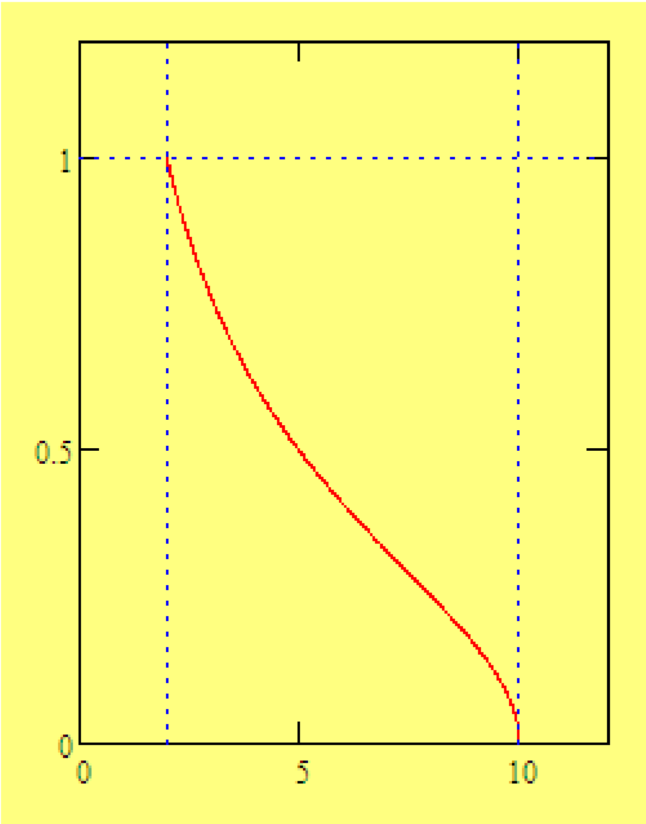
From (2.19) one gets for the surface of the star

$$v_C^g = -\frac{1}{\sqrt{1-\frac{2M}{r'_g}}}\sqrt{\frac{2M}{r'_g}}\sqrt{\frac{1}{\Re}-1} \tag{2.21}$$

and for $v_C^g = -1$ the lowest value for $\Re$. At the beginning of the collapse is $r_g = r'_g$ and therefore is $v_C = 0$. Thus, at $t' = 0$ the scale factor is $\Re = 1$ and one has for it the range of validity

$$\left\{\frac{2M}{r'_g} < \Re \leq 1\right\} . \tag{2.22}$$

Below we have plotted the velocity as function of r on the boundary surface for $r'_g = 10M$. The slung behavior does not agree with a physical progression.



It should also be noted that the collapse velocity which is composed according to Einstein's rules due to (2.20), provides a convincing progression. The use of the Lorentz relations and their integration with respect to the proper time of the freely falling surface would lead to a model whose surface needs an infinitely long time to reach the event horizon. It would correspond to a model called ECO (eternally collapsing object), proposed by Mitra [3]. However, an analytical solution with the ansatz (2.20) for the collapse velocity is not known.

3. THE FIELD QUANTITIES

In [1] we have shown that in the comoving system the Ricci tensor takes on the form

$$R_{m'n'} = -\left['U^{s'}{}_{\underset{1}{\|}s'} + 'U^{s'}\,'U_{s'} \right] h_{m'n'} - \left[B_{n'\underset{2}{\|}m'} + B_{n'}B_{m'} \right] - b_{n'}b_{m'}\left[B^{s'}{}_{\underset{2}{\|}s'} + B^{s'}B_{s'} \right] - \left[C_{n'\underset{3}{\|}m'} + C_{n'}C_{m'} \right] - c_{n'}c_{m'}\left[C^{s'}{}_{\underset{3}{\|}s'} + C^{s'}C_{s'} \right] . \tag{3.1}$$

Therein

$$h_{m'n'} = \begin{pmatrix} 1 & & & \\ & 0 & & \\ & & 0 & \\ & & & 1 \end{pmatrix} \tag{3.2}$$

is a submatrix of the metric.

$$'U_{m'} = 'A_{1'm'}{}^{1'} = \{0,0,0,'U_{4'}\}, \quad 'U_{4'} = \frac{1}{K} K_{|4'} = -\frac{i}{K}\dot{K}, \quad \partial_{4'} = -i\frac{\partial}{\partial t'} \tag{3.3}$$

is the field strength, which describes the collapse and

$$B_{m'} = 'A_{2'm'}{}^{2'} = \left\{ \frac{a_I}{r}, 0, 0, \frac{1}{K} K_{|4'} \right\}, \quad C_{m'} = 'A_{3'm'}{}^{3'} = \left\{ \frac{a_I}{r}, \frac{1}{r}\cot\vartheta, 0, \frac{1}{K} K_{|4'} \right\} \tag{3.4}$$

are the lateral field strengths, the

$$'m_{m'} = \{1,0,0,0\}, \quad b_{m'} = \{0,1,0,0\}, \quad c_{m'} = \{0,0,1,0\}, \quad 'u_{m'} = \{0,0,0,1\} \tag{3.5}$$

are the unit vectors. The meaning of the graded derivatives

$$U_{n\underset{1}{\|}m} = U_{n|m}, \quad B_{n'\underset{2}{\|}m'} = B_{n'|m'} - 'U_{m'n'}{}^{s'}B_{s'}, \quad C_{n'\underset{3}{\|}m'} = C_{n'|m'} - B_{m'n'}{}^{s'}C_{s'} - 'U_{m'n'}{}^{s'}C_{s'} \tag{3.6}$$

is described in [4] in detail.

It can be shown that the field equations in the comoving system have the same form as (3.1), if the graded derivatives are properly defined. Deriving the field equations we cannot rely on the 4-bein system of the non-comoving system, because we have left open the value for a_T.

But we have a powerful method of calculating the field quantities in the non-comoving system, avoiding the metric factors: the Lorentz transformation. Regarding (2.18) and (2.19) we are familiar with the velocity of the particles in the interior of the star, and with the associated Lorentz factor.

The Ricci-rotation coefficients containing the field strengths transform inhomogeneously

$$
\begin{aligned}
{}'A_{m'n'}{}^{s'} &= L^{m\,n\,s'}_{m'n's} A_{mn}{}^{s} + {}'L_{m'n'}{}^{s'}, \quad {}'L_{m'n'}{}^{s'} = L^{s'}_{s} L^{s}_{s'|m'} \\
A_{mn}{}^{s} &= L^{m'n's}_{m\,n\,s'}\, {}'A_{m'n'}{}^{s'} + L_{mn}{}^{s}, \quad L_{mn}{}^{s} = L^{s}_{s'} L^{s'}_{s|m}
\end{aligned} . \tag{3.7}
$$

The last terms in each case we call Lorentz terms. With (2.17) we first compute

$$
\begin{aligned}
{}'L_{4'1'}{}^{4'} &= {}'L_{1'} = i\alpha_C^2 v_{C|4'}, \quad {}'L_{1'4'}{}^{1'} = {}'L_{4'} = -i\alpha_C^2 v_{C|1'} \\
L_{41}{}^{4} &= L_1 = -i\alpha_C^2 v_{C|4}, \quad L_{14}{}^{1} = L_4 = i\alpha_C^2 v_{C|1}
\end{aligned} . \tag{3.8}
$$

The expressions can be clearly arranged:

$$
{}'L_{m'n'}{}^{s'} = h_{m'}{}^{s'}\, {}'L_{n'} - h_{m'n'}\, {}'L^{s'}, \quad L_{mn}{}^{s} = h_m{}^{s} L_n - h_{mn} L^{s} . \tag{3.9}
$$

Intermediate steps are necessary for the calculation of the changes of the velocity. We remember that the collapse of the radius of the spherical cap is a time-dependent variable that enters into this calculation. Since this change of $\mathcal{R}_g$ is determined by the property of the Schwarzschild parabola, we separate the calculated results into a circular part as would be expected with a non-collapsing object and a parabolic part that stems from the collapse. First, one has for the two velocities, which make up the collapse velocity

$$
v^{I}_{|m'} = \{1,0,0,0\}\, a_I v_I \frac{1}{r}, \quad v^{I}_{|m} = \{\alpha_C, 0, 0, -i\alpha_C v_C\}\, a_I v_I \frac{1}{r} , \tag{3.10}
$$

$$
\begin{aligned}
v^{R}_{|m'} &= \{\alpha_C, 0, 0, -i\alpha_C v_C\}\left(-\frac{a_R}{\mathcal{R}_g}\right) + \{0,0,0,1\}\left(-3i\alpha_C v_C \frac{a_R}{\rho_g}\right) \\
v^{R}_{|m} &= \{1,0,0,0\}\left(-\frac{a_R}{\mathcal{R}_g}\right) + \{-i\alpha_C v_C, 0, 0, \alpha_C,\}\left(-3i\alpha_C v_C \frac{a_R}{\rho_g}\right)
\end{aligned} . \tag{3.11}
$$

With some computational effort it follows from (2.19)

$$
\begin{aligned}
v^{C}_{|m'} &= \frac{1}{\alpha_C^2}\{i\alpha_C v_C, 0, 0, \alpha_C\}\, i\alpha_R v_R \frac{1}{\mathcal{R}_g} + \frac{1}{\alpha_C}\{0,0,0,1\}\left(-3i\alpha_R v_R \frac{1}{\rho_g}\right) \\
&\quad + \frac{1}{\alpha_C}\{1,0,0,0\}\, v_C \frac{a_R}{r} \\
v^{C}_{|m} &= \frac{1}{\alpha_C^2}\{0,0,0,1\}\, i\alpha_R v_R \frac{1}{\mathcal{R}_g} + \frac{1}{\alpha_C}\{-i\alpha_C v_C, 0, 0, \alpha_C\}\left(-3i\alpha_R v_R \frac{1}{\rho_g}\right) \\
&\quad + \frac{1}{\alpha_C}\{\alpha_C, 0, 0, i\alpha_C v_C\}\, v_C \frac{a_R}{r}
\end{aligned} \tag{3.12}
$$

and finally with (3.8)

$$
\begin{aligned}
L^{C}_{m} &= \{1,0,0,0\}\left(\alpha_R v_R \frac{1}{\mathcal{R}_g}\right), \quad L^{P}_{m} = \{\alpha_C, 0, 0, i\alpha_C v_C\}\left(-3\alpha_C \alpha_R v_R \frac{1}{\rho_g}\right) \\
{}'L^{C}_{m'} &= \{\alpha_c, 0, 0, -i\alpha_C v_C\}\left(-\alpha_R v_R \frac{1}{\mathcal{R}_g}\right), \quad {}'L^{P}_{m'} = \{1,0,0,0\}\left(3\alpha_C \alpha_R v_R \frac{1}{\rho_g}\right)
\end{aligned} . \tag{3.13}
$$

For the whole Lorentz term one obtains

$$L_m = L^C_{m'} + L^P_m - {}'U_m, \quad {}'L_{m'} = {}'L^C_{m'} + {}'L^P_{m'} + {}'U_{m'}, \quad {}'U_m = L^{m'}_m {}'U_{m'} \ . \tag{3.14}$$

With this we have the tools in hand to calculate the missing quantities of the non-comoving system. From the inhomogeneous transformation law (3.7) and (3.9) one gets the simple relations

$$'U_{m'} = U_{m'} + {}'L_{m'}, \quad U_m = {}'U_m + L_m . \tag{3.15}$$

Now we are able to assort all the components of the U-quantities

$$\begin{aligned} &U^C_m = \{1,0,0,0\}\,\alpha_R v_R \frac{1}{\mathcal{R}_g}, \qquad U^P_m = \{\alpha_C, 0, 0, i\alpha_C v_C\}\left(-3\alpha_C\alpha_R v_R \frac{1}{\rho_g}\right) \\ &U^C_{m'} = \{\alpha_C, 0, 0, -i\alpha_C v_C\}\,\alpha_R v_R \frac{1}{\mathcal{R}_g}, \quad U^P_{m'} = \{1,0,0,0\}\left(-3\alpha_C\alpha_R v_R \frac{1}{\rho_g}\right) \end{aligned} , \tag{3.16}$$

whereby we have again made the decomposition into a circular and a parabolic part. We also recognize that the U-variables are already included in the Lorentz terms

$$'L^C_{m'} = -U^C_{m'}, \quad {}'L^P_{m'} = -U^P_{m'}, \quad L^C_m = U^C_m, \quad L^P_m = U^P_m \ . \tag{3.17}$$

The lateral field quantities transform as vectors

$$B_m = L^{m'}_m B_{m'}, \quad C_m = L^{m'}_m C_{m'} \ . \tag{3.18}$$

Thus, we have shown that it is possible to calculate all field quantities of the non-comoving system without complete knowledge of the metric in the non-comoving co-ordinate system. Now the question arises whether the metric coefficients of the non-comoving system can be deduced from the previous results. In the above calculations we have repeatedly relied on the cap of a sphere as basic geometric structure and have written down a corresponding metric (2.5), (2.6), and (2.10). The space-like part of the metric is well known from other models, also the assumption that the radial metric factor (2.6) corresponds to the Lorentz factor of a motion $v_R = -r/\mathcal{R}_g$.

With (2.18) and using (2.15) the components of B in (3.4) may be brought into the form

$$B_{m'} = \{\alpha_C, 0, 0, -i\alpha_C v_C\}\frac{a_R}{r} , \tag{3.19}$$

whereby the familiar structures of the spherical geometry

$$B_m = \left\{\frac{a_R}{r}, 0, 0, 0\right\}, \quad C_m = \left\{\frac{a_R}{r}, \frac{1}{r}\cot\vartheta, 0, 0\right\} \tag{3.20}$$

can be obtained for (3.18). Since

$$B_1 = \frac{1}{r} r_{|1} = \frac{1}{r} e^1_{\ 1} \frac{\partial}{\partial r} r = \frac{a_R}{r}$$

$\underset{1}{e}^1 = a_R$, $\overset{1}{e}_1 = \alpha_R = 1/a_R$ must be valid. Therefore the cap of a sphere is a suitable object to give the model a geometric basis. To approach the outstanding time-like metric factor, we calculate with $\frac{1}{\alpha}\alpha_{|1} = \alpha^2 v v_{|1}$ and (3.11)

$$\frac{1}{\alpha}\alpha_{|1} = -\alpha_R v_R \frac{1}{\rho_g} - 3\alpha_C^2 v_C^2 \alpha_R v_R \frac{1}{\rho_g} \ . \tag{3.21}$$

Using the relation $\rho_g = 2\mathcal{R}_g$ one can write the quantity U_1 in the form

$$U_1 = -\alpha_R v_R \frac{1}{\mathcal{R}_g} - 3\alpha_C^2 v_C^2 \alpha_R v_R \frac{1}{\rho_g} + \alpha_R v_R \frac{1}{\rho_g} \ . \tag{3.22}$$

It follows

$$U_1 = \frac{1}{a_T} a_{T|1} = \frac{1}{\alpha}\alpha_{|1} + \alpha_R v_R \frac{1}{\rho_g} \ . \tag{3.23}$$

If one cannot express the last term as a gradient the non-comoving co-ordinate system is anholonomic. Then no relation can be specified between the co-ordinate times t and t'. From this example one can see the difficulty in finding suitable co-ordinate systems for collapsing models. By no means should one imagine that a model can be represented as a 4-dimensional surface in a flat higher dimensional space, whereby the surface is covered by a Gaussian co-ordinate system, and one of these co-ordinates is the time co-ordinate.

As regards the exterior Schwarzschild solution the space-like part of Flamm's paraboloid still fulfills our traditional concepts of the embedding of surfaces into a higher dimensional flat space. The time-like part of the metric needs a sixth variable, whereby two of these variables lie in one and the same dimension, so that the embedding into the 5-dimensional flat space can be sustained.

The complexities of the time-like part of the metric are to be taken into account if the interior solution of a collapsing star is to be linked to the exterior Schwarzschild field. This is a challenge for anyone who deals with this field of problems.

4. THE STRESS-ENERGY-MOMENTUM TENSOR

The Ricci in the non-comoving system has the same shape as in the comoving system

$$\begin{aligned} R_{mn} = &-\left[U^s{}_{\underset{1}{||}s} + U^s U_s\right] h_{mn} \\ &-\left[B_{n\underset{2}{||}m} + B_n B_m\right] - b_n b_m \left[B^s{}_{\underset{2}{||}s} + B^s B_s\right] \\ &-\left[C_{n\underset{3}{||}m} + C_n C_m\right] - c_n c_m \left[C^s{}_{\underset{3}{||}s} + C^s C_s\right] \end{aligned} , \tag{4.1}$$

if one appropriately defines the graded derivatives

$$U_{m\underset{1}{||}n} = U_{m|n}, \quad B_{m\underset{2}{||}n} = B_{m|n} - U_{nm}{}^s B_s, \quad C_{m\underset{3}{||}n} = C_{m|n} - U_{nm}{}^s C_s - B_{nm}{}^s C_s \ . \tag{4.2}$$

Therein is

$$U_{mn}{}^s = h_m{}^s U_n - h_{mn} U^s . \tag{4.3}$$

In the previous Section we have calculated the field quantities by means of a Lorentz transformation. The stress-energy-momentum tensor of the non-comoving system can also be calculated with a Lorentz transformation

$$T_{mn} = L_{m\ n}^{m'n'} T_{m'n'} . \tag{4.4}$$

Since the stress-energy-momentum tensor in the comoving system has the only component

$$T_{4'4'} = \mu_0, \quad \kappa\mu_0 = \frac{3}{\mathcal{R}_g^2} \tag{4.5}$$

one obtains for the non-comoving system

$$T_{11} = -\alpha_C^2 v_C^2 \mu_0, \quad T_{14} = -i\alpha_C^2 v_C \mu_0, \quad T_{44} = \alpha_C^2 \mu_0 \ . \tag{4.6}$$

In particular, we are here interested in how the components of the stress-energy-momentum tensor arise from the geometric components of the Einstein tensor. First, we calculate the quantities

$$\begin{gathered} \frac{1}{\alpha_R} \alpha_{R|m} = E_m^C + E_m^P \\ E_m^C = \{1,0,0,0\}\left(-\alpha_R v_R \frac{1}{\mathcal{R}_g}\right), \quad E_m^P = \{-i\alpha_C v_C, 0, 0, \alpha_C\}\left(-3i\alpha_C v_C \alpha_R v_R \frac{1}{\rho_g}\right) \end{gathered} . \tag{4.7}$$

With the parabolic part of the above relation and with U_m^P from (3.6) we finally obtain the desired relations

$$\left(B_1 + C_1\right)E_1^P = \alpha_C^2 v_C^2 \frac{3}{\mathcal{R}_g^2} = \alpha_C^2 v_C^2 \kappa\mu_0 = -\kappa T_{11}$$

$$\left(B_1 + C_1\right)E_4^P = i\alpha_C^2 v_C \frac{3}{\mathcal{R}_g^2} = i\alpha_C^2 v_C \kappa\mu_0 = -\kappa T_{14} \quad , \tag{4.8}$$

$$\left(B_1 + C_1\right)U_1^P = -\alpha_C^2 \frac{3}{\mathcal{R}_g^2} = -\alpha_C^2 \kappa\mu_0 = -\kappa T_{44}$$

whereby the remaining terms of the Einstein tensor are canceled. Thus one has worked out an interesting structure of the field equations of collapsing stars.

5. SUMMARY

We have geometrically deepened an understanding of the model for a collapsing star of Weinberg. We have found general structures which may be helpful for the construction of other models.

6. LITERATURE

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