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REMARKS ON THE MODEL OF WEINBERG

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Contents

Abstract: We reinvestigate the collapsing model of Weinberg, we calculate the physical component of the collapse velocity, and we study the behavior of the surface of the collapsing object at the event horizon.

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1. INTRODUCTION

In his textbook Weinberg [1] has presented a time-dependent solution which describes a gravitational collapse of dust. The stellar object collapses towards its center at $r = 0$ and shrinks to a point in finite proper time. We will show that there is an inconsistency concerning the calculation of the collapse velocity. By using Weinberg's results we will evaluate the Lorentz transformation connecting a comoving and a non-comoving reference system and we will read off the collapse velocity from this Lorentz transformation. It turns out that the velocity of the surface of the collapsing object would reach the velocity of light at the event horizon.

2. COLLAPSE IN THE COMOVING CO-ORDINATE SYSTEM

Weinberg starts with an ansatz describing dust consisting of incoherent matter with internal pressure $p = 0$. Thus, the stress-energy tensor has the simple form

$$T_{mn} = \mu_0 u_m u_n, \quad \mu_0 = \mu_0(t') \ . \tag{2.1}$$

We use a comoving co-ordinate system with the radial comoving co-ordinate r' and the comoving time t'. In this system the 4-velocity has the components

$$u_m = \{0,0,0,1\} \ . \tag{2.2}$$

Due to the requirement for spherical symmetry and the choice of appropriate initial conditions Weinberg has obtained a collapsing interior solution which he has adapted somewhat grindingly to the exterior Schwarzschild solution. Since the solution is known to us, we can write the metric in a form which allows us a better understanding of the model

$$ds^2 = \mathcal{K}^2 \left[\frac{1}{1 - \frac{r'^2}{\mathcal{R}_0^2}} dr'^2 + r'^2 d\vartheta^2 + r'^2 \sin^2\vartheta \, d\varphi^2 \right] - dt'^2 . \tag{2.3}$$

In this Section we follow the derivation of Weinberg, but we use the tetrad representation instead. The geometrical background of the space-like part of the metric is the cap of a sphere with the initial radius $\mathcal{R}_0 = \text{const.}$ and the polar angles ϑ and φ. $\mathcal{K} = \mathcal{K}(t')$ is a time-dependent dimensionless quantity, describing the collapse. From the metric (2.3) we read the tetrads

$$\overset{1}{e}_1 = \mathcal{K}\alpha_I, \quad \overset{2}{e}_2 = \mathcal{K} r', \quad \overset{3}{e}_3 = \mathcal{K} r' \sin\vartheta, \quad \overset{4}{e}_4 = 1, \quad \alpha_I = \frac{1}{\sqrt{1 - \frac{r'^2}{\mathcal{R}_0^2}}}, \quad a_I = \frac{1}{\alpha_I} \tag{2.4}$$

and we calculate the Ricci-rotation coefficients

$$A_{mn}{}^{s} = U_{mn}{}^{s} + B_{mn}{}^{s} + C_{mn}{}^{s}$$
$$U_{mn}{}^{s} = m_m U_n m^s - m_m m_n U^s, \quad B_{mn}{}^{s} = b_m B_n b^s - b_m b_n B^s, \quad C_{mn}{}^{s} = c_m C_n c^s - c_m c_n C^s \tag{2.5}$$

with the unit vectors

$$m_m = \{1,0,0,0\}, \quad b_m = \{0,1,0,0\}, \quad c_m = \{0,0,1,0\}, \quad u_m = \{0,0,0,1\} \ . \tag{2.6}$$

The field strengths are

$$U_m = \{0,0,0,U_4\}, \quad U_4 = \frac{1}{K} K_{|4} = -\frac{i}{K} \dot{K}, \quad \partial_4 = -i\frac{\partial}{\partial t'}$$
$$B_m = \left\{\frac{a_I}{Kr'}, 0, 0, \frac{1}{K} K_{|4}, \right\}, \quad C_m = \left\{\frac{a_I}{Kr'}, \frac{1}{Kr'} \cot\vartheta, 0, \frac{1}{K} K_{|4}, \right\} \ , \tag{2.7}$$

wherein $U_4 \stackrel{*}{=} B_4 \stackrel{*}{=} C_4$ is a relation between the time-like quantities which we will use to simplify some calculations. With the expressions mentioned above, the Ricci has the form

$$\begin{aligned} R_{mn} = &-\left[U^s{}_{\underset{1}{||}s} + U^s U_s\right] h_{mn} \\ &-\left[B_{n\underset{2}{||}m} + B_n B_m\right] - b_n b_m \left[B^s{}_{\underset{2}{||}s} + B^s B_s\right] \ , \\ &-\left[C_{n\underset{3}{||}m} + C_n C_m\right] - c_n c_m \left[C^s{}_{\underset{3}{||}s} + C^s C_s\right] \end{aligned} \tag{2.8}$$

$$h_{mn} = \begin{pmatrix} 1 & & & \\ & 0 & & \\ & & 0 & \\ & & & 1 \end{pmatrix} . \tag{2.9}$$

The graded derivatives [7] are defined by

$$U_{n\underset{1}{||}m} = U_{n|m}, \quad B_{n\underset{2}{||}m} = B_{n|m} - U_{mn}{}^{s} B_s, \quad C_{n\underset{3}{||}m} = C_{n|m} - B_{mn}{}^{s} C_s - U_{mn}{}^{s} C_s \ . \tag{2.10}$$

Having calculated (2.8) with the help of (2.10) and using the definition

$$'g_{mn} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 0 \end{pmatrix} \tag{2.11}$$

the Einstein tensor is reduced to

$$G_{mn} = \left[2U_{4|4} + 3U_4 U_4 - \frac{1}{K^2 R_0^2}\right] 'g_{mn} + 3\left[U_4 U_4 - \frac{1}{K^2 R_0^2}\right] u_m u_n = -\kappa \mu_0 u_m u_n \ . \tag{2.12}$$

This relation decomposes into the two equations

$$2U_{4|4} + 3U_4 U_4 - \frac{1}{K^2 R_0^2} = 0, \quad 3\left[U_4 U_4 - \frac{1}{K^2 R_0^2}\right] = -\kappa \mu_0 \tag{2.13}$$

which have to be solved. Using $K_{|4} = -iK^{\bullet}$ the latter can be written as

$$3\left[\frac{1}{K^2}K^{\bullet 2} + \frac{1}{K^2 R_0^2}\right] = \kappa\mu_0 .$$

At the beginning of the collapse at the time $t' = 0$ one has

$$K^{\bullet}(0) = 0, \quad K(0) = 1.$$

From the expressions mentioned above follows

$$\kappa\mu_0(0) = \frac{3}{R_0^2} , \qquad (2.14)$$

a relation which is known from other models. Let us take a look at the conservation law which is reduced due to the simplicity of (2.1) to

$$T^m{}_{4||m} = T^4{}_{4|4} + A_m T^m{}_4 = \mu_{0|4} + 3U_4\mu_0 = 0 . \qquad (2.15)$$

This becomes

$$\frac{1}{\mu_0}\mu_0^{\bullet} + 3\frac{1}{K}K^{\bullet} = 0, \quad \left(\ln\mu_0 K^3\right)^{\bullet} = 0$$

and from this one gets the important relation

$$\mu_0 K^3 = \text{const.} \; . \qquad (2.16)$$

Multiplying the above equation by K^3 one gets

$$3\left[KK^{\bullet 2} + \frac{K}{R_0^2}\right] = \kappa\mu_0 K^3 . \qquad (2.17)$$

This result can be processed with

$$\mu_0(t')K^3(t') = \mu_0(0)K^3(0) = \mu_0(0) = \text{const.}.$$

Inserting into (2.17) we obtain with (2.14) the differential equation

$$K^{\bullet 2} = \frac{1}{R_0^2}\left(\frac{1}{K} - 1\right), \quad K^{\bullet} = -\frac{1}{R_0}\sqrt{\frac{1}{K} - 1} \qquad (2.18)$$

after some reshaping. Now the first relation of (2.13) is to be treated. Since

$$U_{4|4} = -\frac{1}{K}K^{\bullet\bullet} + \frac{1}{K^2}K^{\bullet}K^{\bullet}$$

results, and if multiplied by K^2, one has from (2.13)

$$2KK^{\bullet\bullet} + K^{\bullet 2} + \frac{1}{R_0^2} = 0 .$$

If one uses (2.18) one arrives at the second-order differential equation

$$K^{\bullet\bullet} = -\frac{1}{2K^2 R_0^2} . \qquad (2.19)$$

It has a solution given by the parameter equations

$$t' = \frac{\mathcal{R}_0}{2}(\psi + \sin\psi), \quad \mathcal{K} = \frac{1}{2}(1+\cos\psi). \tag{2.20}$$

Eliminating ψ leads to

$$t' = \mathcal{R}_0\left[\frac{1}{2}\arccos(2\mathcal{K}-1) + \sqrt{\mathcal{K}(1-\mathcal{K})}\right]. \tag{2.21}$$

We supplement Weinberg's calculus with the quantity

$$\mathcal{R} = \sqrt{\mathcal{K}^3}\,\mathcal{R}_0 \;. \tag{2.22}$$

We will discuss this definition later in more detail. $\mathcal{R}_0$ is the initial value for the radius of a spherical geometry which geometrically describes the collapsing object, and $\mathcal{R}$ is the radius of the sphere at any given time. For $\mathcal{K}=1$ one obtains $\psi=0$ and thus $t'=0$, the initial state of collapse with the initial value $\mathcal{R}=\mathcal{R}_0$. For $\mathcal{K}=0$ and $\psi=\pi$ one has $t' = \frac{\pi}{2}\mathcal{R}_0$ and $\mathcal{R}=0$. The radius of the spherical cap and thus the whole cap of the sphere has shrunk to a point. This can be seen in Fig. 1.

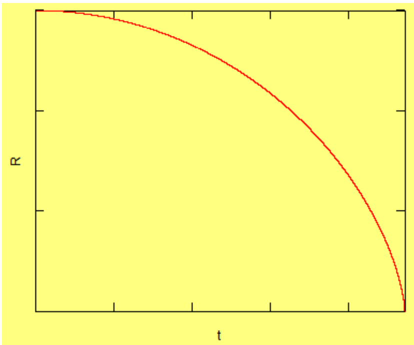

Fig.1

The stellar object collapses after a finite proper time $T' = (\pi/2)\mathcal{R}_0$ from rest to a state of infinitely high energy density, as equation

$$\mu_0 = \frac{\mu_0(0)}{\mathcal{K}^3}$$

with $\mathcal{K}(T')=0$ shows. Other models also have this property, which is physically less interpretable. With (2.14) and (2.22) one finally has

$$\kappa\mu_0 = \frac{3}{\mathcal{R}^2} \tag{2.23}$$

for an arbitrary time during the collapse.

Redifferentiation of the first equation (2.20) leads to

$$dt' = \frac{\mathcal{R}_0}{2}(1+\cos\psi)\,d\psi = \mathcal{K}\mathcal{R}_0\,d\psi \;, \tag{2.24}$$

the approach commonly used by us for the time-like arc element. In addition, we note

$$r' = \mathcal{R}_0 \sin\eta' , \tag{2.25}$$

wherein the radial co-ordinate r' and the polar angle η' are time-independent quantities in the comoving co-ordinate system. With this ansatz the metric (2.3) can be brought into the form

$$ds^2 = \mathcal{K}^2\left[\mathcal{R}_0^2 d\eta'^2 + \mathcal{R}_0^2 \sin^2\eta' d\vartheta^2 + \mathcal{R}_0^2 \sin^2\eta' \sin^2\vartheta d\varphi^2\right] - dt'^2 . \tag{2.26}$$

If one inserts the relation (2.24) one finds

$$ds = \mathcal{K}(t') ds_0 , \tag{2.27}$$

wherein ds_0 is the line element of the spherical space at the time $t' = 0$.

A comparison with the Friedman cosmological solution of the Section II.5 of our paper [7] shows the close formal relationship between the two models. If one puts in formula (II.5.14) of this paper the integration constant to

$$a = \frac{3}{\kappa} \frac{1}{\mathcal{R}_0^2}$$

and the cosmological constant $\lambda = 0$, and if one now writes $\mathcal{R}$ instead of $\mathcal{K}$, then the differential equation of Friedman (II.5.15) is reduced to (2.19). If we had written the metric in the form (2.26) right away, we would have been able to rely on the Friedman solution and we would have been able to convert the expansion into a contraction by an appropriate choice of sign.

It may be advantageous to combine the time-like quantities and to split them off from the field equations. Utilizing tetrad methods, one obtains from (2.3) the Ricci-rotation coefficients in the form

$$A_{mn}{}^{s} = {}^*A_{mn}{}^{s} + {}'\delta_m^s U_n - {}'g_{mn} U^s, \quad A_m = {}^*A_m + 3U_m , \tag{2.28}$$

wherein $^*A$ includes the space-like but time-dependent field strengths B and C of the lateral part of the metric. $'g_{mn}$ is the 3-dimensional space-like part of the metric tensor and $^*A_m = {}^*A_{sm}{}^{s}$. For the Ricci with respect to $^*A$ only remains

$$^*R_{mn} = \frac{2}{\mathcal{K}^2 \mathcal{R}_0^2} {}'g_{mn} .$$

3. COLLAPSE IN THE NON-COMOVING CO-ORDINATE SYSTEM

In his book Weinberg has also noted the metric of the collapsing object in non-comoving co-ordinates. This results in the 4-bein

$$\overset{1}{e}_1 = \frac{1}{\sqrt{1 - \frac{r'^2}{\mathcal{R}_0^2}\frac{1}{\mathcal{K}}}}, \qquad \overset{4}{e}_4 = \sqrt{\frac{\mathcal{K}}{S}}\sqrt[4]{\frac{1 - r'^2/\mathcal{R}_0^2}{1 - r_g'^2/\mathcal{R}_0^2}}\frac{1 - \frac{r'^2}{\mathcal{R}_0^2}\frac{1}{S}}{\sqrt{1 - \frac{r'^2}{\mathcal{R}_0^2}\frac{1}{\mathcal{K}}}} \ . \tag{3.1}$$

$$S = 1 - \sqrt{\frac{1 - r'^2/\mathcal{R}_0^2}{1 - r_g'^2/\mathcal{R}_0^2}}\left(1 - \mathcal{K}\right)$$

r_g' denotes the value of r' on the surface of the stellar object. Since it is assumed that the collapsing star produces a Schwarzschild field around itself which is static due to the Birkhoff theorem, the values (3.1) have to match on the surface of the stellar object with the Schwarzschild values.

$\mathcal{R}_0$ is not only the radius of curvature of the spherical cap at the beginning of the collapse, which defines the geometry of the model of Weinberg. However, it has a meaning at the boundary surface of the interior and exterior geometries concerning the Schwarzschild parabola. The Schwarzschild geometry is usually treated in non-comoving co-ordinates, which we denote by {r, t}.

If one extends the radius of curvature of the Schwarzschild parabola

$$\rho = \sqrt{\frac{2r^3}{M}}$$

to the directrix of the Schwarzschild parabola, then the distance $\mathcal{R}$ is cut out. It has half the length of ρ. Thus, one has

$$\mathcal{R}_g = \sqrt{\frac{r_g^3}{2M}} \tag{3.2}$$

on the boundary surface of the geometries. At the beginning of the collapse at $t' = 0$ one has $r_g' = r_g$, thus

$$\mathcal{R}_g(0) = \mathcal{R}_0 = \sqrt{\frac{r_g'^3}{2M}} \tag{3.3}$$

applies. Inserting this into (3.1) we indeed obtain the Schwarzschild values of the metric coefficients

$$\overset{1}{e}_1 = \frac{1}{\sqrt{1 - \frac{2M}{r_g}}} = \alpha_g^E, \quad \overset{4}{e}_4 = \sqrt{1 - \frac{2M}{r_g}} \ . \tag{3.4}$$

If one still keeps in mind that the lateral part of the metric must be invariant, it follows from (2.26)

$$\mathcal{K}^2 r'^2 d\vartheta^2 + \mathcal{K}^2 r'^2 \sin^2\vartheta d\varphi^2 = r^2 d\vartheta^2 + r^2 \sin^2\vartheta d\varphi^2$$

and from this

$$r = \mathcal{K} r' . \tag{3.5}$$

At this point we want to substantiate the relation (2.22). From (3.2) one obtains

$$\mathcal{R}_g = \sqrt{\frac{\mathcal{K}_g^3 r'^3_g}{2M}}, \quad \mathcal{R}_g = \sqrt{\mathcal{K}_g^3}\,\mathcal{R}_0 \tag{3.6}$$

with the above relation. Since this relation is valid not only on the boundary of the geometry but also in the interior of the stellar object, (2.22) is justified. However, the relation (3.3) is not transferable to the interior.

The invariance of the lateral arc elements is also applicable to the polar representation of the spherical cap. From

$$r d\vartheta = \mathcal{K}\mathcal{R}_0 \sin\eta' d\vartheta = \mathcal{R} \sin\eta d\vartheta$$

with η as a non co-moving polar angle one obtains

$$\sin\eta' = \sqrt{\mathcal{K}} \sin\eta \tag{3.7}$$

with (2.22). From this one gets

$$\cos^2\eta = 1 - \frac{1}{\mathcal{K}} \sin^2\eta' = 1 - \frac{r'^2}{\mathcal{R}_0^2}\frac{1}{\mathcal{K}} .$$

For the non-comoving bein vector

$$\overset{1}{e}_1 = \frac{1}{\cos\eta} = \frac{1}{\sqrt{1 - \frac{r'^2}{\mathcal{R}_0^2}\frac{1}{\mathcal{K}}}} = \frac{1}{\sqrt{1 - \frac{r^2}{\mathcal{R}^2}}}$$

a value results which coincides with the one of Weinberg (3.1).

After these considerations we are able to determine the velocity of the collapse of the stellar object by calculating the velocity on its surface. From (3.5) and

$$dr = \mathcal{K} dr' + \mathcal{K}^{\bullet} r' dt', \quad \mathcal{K}^{\bullet} = \frac{\partial \mathcal{K}}{\partial t'}$$

the coefficients of the co-ordinate transformation can be calculated as

$$\Lambda^i_{i'} = x^i_{|i'} . \tag{3.8}$$

With (2.18) and $\partial_{4'} = -i\frac{\partial}{\partial t'}$ one has

$$\Lambda^1_{1'} = \mathcal{K}, \quad \Lambda^1_{4'} = i\frac{r'}{\mathcal{R}_0}\sqrt{\frac{1}{\mathcal{K}} - 1} . \tag{3.9}$$

However, on the boundary of the surface the second term yields together with (3.3)

$$\Lambda^1_{4'} = i\sqrt{\frac{2M}{r'_g}}\sqrt{\frac{1}{\mathcal{K}} - 1} = i\sqrt{\frac{2M}{r_g} - \frac{2M}{r'_g}} ,$$

whereby (3.5) has been used in the last step.

To get the *physical* value of the collapse velocity, the Lorentz transformation must be determined by transvecting the Λ with the 4-beine

$$L^{m}_{m'} = \overset{m}{e}_{i}\,\Lambda^{i}_{i'}\,\underset{m'}{e}^{i'} \quad . \tag{3.10}$$

This gives on the boundary surface

$$L^{1}_{1'} = \alpha^{g}_{E}\cdot\mathcal{K}_g\cdot\frac{1}{\mathcal{K}_g}\sqrt{1-\frac{2M}{r'_g}} = \frac{\sqrt{1-\frac{2M}{r'_g}}}{\sqrt{1-\frac{2M}{r_g}}}, \quad L^{1}_{4'} = \alpha^{g}_{E}\cdot i\sqrt{\frac{2M}{r_g}-\frac{2M}{r'_g}}\cdot 1 = i\frac{\sqrt{\frac{2M}{r_g}-\frac{2M}{r'_g}}}{\sqrt{1-\frac{2M}{r_g}}} \quad . \tag{3.11}$$

By putting this result into the Lorentz form

$$L^{1}_{1'} = \alpha_{col}, \quad L^{1}_{4'} = -i\alpha_{col}v_{col}$$
$$\alpha_{col} = \frac{\sqrt{1-\frac{2M}{r'_g}}}{\sqrt{1-\frac{2M}{r_g}}}, \quad v_{col} = -\frac{\sqrt{\frac{2M}{r_g}-\frac{2M}{r'_g}}}{\sqrt{1-\frac{2M}{r'_g}}} \quad , \tag{3.12}$$

we have identified the collapse velocity with v_{col}. Therein $2M/r'_g$ is a constant quantity. The initial velocity is $v_{col}=0$ at the time $t'=0$ as a consequence of $r_g=r'_g$ and the Lorentz factor is $\alpha_{col}=1$. If the surface of the stellar object has reached the event horizon of the Schwarzschild geometry at $r_g=2M$, then we get $v_{col}(r_g=2M)=-1$ and $\alpha_{col}=\infty$. The collapse has achieved the speed of light in free fall and the surface having crossed the event horizon would move faster than light. The force of gravity first gets infinite and then would get imaginary.

Weinberg also relates the co-ordinate times of the two systems:

$$t = \mathcal{R}_0\sqrt{1-\frac{r'^2_g}{\mathcal{R}^2_0}}\int_{S(r',t')}^{1}\frac{1}{1-\frac{r'^2_g}{\mathcal{R}^2_0}\frac{1}{\mathcal{K}}}\sqrt{\frac{\mathcal{K}}{1-\mathcal{K}}}d\mathcal{K}, \quad d\mathcal{K} = \dot{\mathcal{K}}dt' = -\frac{1}{\mathcal{R}_0}\sqrt{\frac{1}{\mathcal{K}}-1}dt' \quad . \tag{3.13}$$

The integral has the solution

$$t = -\mathcal{R}_0\sqrt{1-A}\left[\frac{2A+1}{2}\arcsin\left(1-2\mathcal{K}\right)+\sqrt{\mathcal{K}\left(1-\mathcal{K}\right)}+2\sqrt{\frac{A^3}{1-A}}\,\mathrm{ar\,th}\sqrt{\frac{\mathcal{K}}{A}\frac{1-A}{1-\mathcal{K}}}\right]^{1}_{S(r',t')}, \tag{3.14}$$
$$A = \frac{r'^2_g}{\mathcal{R}^2_0}$$

whereby the quantity S simplifies to $S_g = \mathcal{K}_g(t')$ on the surface of the star. On this Weinberg takes up further consideration. A light signal which is radially emitted from the surface of the collapsing star needs an infinitely long co-ordinate time in order to reach an observer when the surface coincides with the event horizon. This observer sees the star collapsing eternally. But below the event horizon the star cannot be observed by him. Due to the increasing redshift, he sees the star fade slowly when the surface of the star is approaching the event horizon.

However, we do not find this scenario described by Weinberg realistically. On the one hand, the star collapses in a relatively short proper time to a point with an infinite mass density. On the other hand, for an outside observer this star eternally has a finite extension. In fact, two observers, who are in different states of motion, have different views of the chronological sequences according to the principle of relativity. The principle of relativity does not go so far that an object can have two different, mutually exclusive states. Therefore, we suggest an inconsistency in the model of Weinberg.

Once again we look at the formulae (3.12) and we interpret them. Then

$$v = -\sqrt{\frac{2M}{r_g}} \tag{3.15}$$

is the velocity of an observer who is in free fall coming from infinity and is located at the very position $r_g \le r'_g$.

$$r_0 = r'_g = r_g\left(t' = 0\right) \tag{3.16}$$

is the position of an observer who is at the time $t' = 0$, i.e. at the start of the collapse at the position r_0. Thus,

$$v_0 = -\sqrt{\frac{2M}{r'_g}} \tag{3.17}$$

is the velocity of an observer coming from the infinite who is just passing the position r_0. As we have stated in previous papers [2-6], the fall velocity v of an object that comes from infinity can be directly calculated. But the speed v' of an observer who falls away from r_0 can only be calculated circuitously by using the relativistic difference of v and v_0. From Einstein's law of the addition of velocities results

$$v' = \frac{v - v_0}{1 - vv_0} \ . \tag{3.18}$$

This should also apply to the surface of a star. A glance at (3.12) shows that

$$v_{col} = -\frac{\sqrt{v^2 - v_0^2}}{\sqrt{1 - v_0^2}} \tag{3.19}$$

has a different structure and contradicts the relativistic laws. For the associated Lorentz factor, we would expect

$$\alpha' = \alpha\alpha_0\left(1 - vv_0\right) \ . \tag{3.20}$$

However, in (3.12) we have

$$\alpha_{col} = \frac{\alpha}{\alpha_0} \ , \tag{3.21}$$

a term which has, according to the Lorentz equations, the ratio

$$\frac{dT}{dT'} = \frac{\alpha}{\alpha_0} \tag{3.22}$$

of the proper times of the observer in rest and the observer falling away from r_0, but related to a system that is in free fall coming from *infinity*, but not from r_0, which would be correct.

The formulae (3.12) are those that correspond to the formulae of free fall in the textbook by Misner, Thorne, and Wheeler [8] and are incorrect, as we have stated in previous papers [2-6]. MTW have achieved these formulae by using a not appropriate expression in the line element. But how did the wrong formulae in the model of Weinberg come about? The cause is to be found in the ansatz (2.3) for the line element. The first three terms of the line element are written in co-ordinates that are comoving with the particles inside the collapsing object. In particular, the movement of the surface starts at the time $t' = 0$ at the position r_0 according to (3.16). However, the time-like part of the line element with dt' refers to a co-ordinate system that is comoving with an observer who is in free fall from *infinity*. It should be written as dt'' to mark the meaning of the term correctly.

We will discuss this last statement in more detail. If one has three observers, one of them rests in the Schwarzschild field, the second is released from r_0, and the third comes from infinity, each observer has his specific proper time and Lorentz factor, namely

$$dT = \alpha a_S dt, \quad dT' = \alpha' a_S dt', \quad dT'' = \alpha'' a_S dt'', \quad a_S = \sqrt{1 - \frac{2M}{r}} \,.$$

For the observer at rest is $\alpha = 1$ because $v = 0$. However, for the observer who comes from infinity one has $\alpha'' = 1/a_S$. Finally one has

$$dT = a_S dt, \quad dT' = \alpha' a_S dt', \quad dT'' = dt'', \quad a_S = \sqrt{1 - \frac{2M}{r}} \,. \tag{3.23}$$

In our paper [4] we have justified these relations with Lemaître-transformations. Only in the system which comes in free fall from infinity, the proper time T" coincides with the co-ordinate time t'. For an observer who falls away from an arbitrary position the redshift factor is different from 1. For this reason we could have been able at the very beginning to state that the Weinberg model is inconsistent.

The fact that many authors start with a line element for a collapsing model with the redshift factor 1 is probably due to the fact that they rely on existing spherically symmetric cosmological solutions, most of which originate from Tolman. In these solutions the infinity is of less importance, and there is no need to match the Schwarzschild solution.

4. SUMMARY

We have shown that the surface of a collapsing star is subjected to the action of the exterior Schwarzschild field. If the stellar object has contracted near to the event horizon, strong gravitational forces will act on the star. As it consists of pressureless dust, it would not resist the forces which would be infinitely high at the event horizon. Thus, we have doubts concerning the consistency of the model.

5. REFERENCES

[1] Weinberg S., *Gravitation and Cosmology.* John Wiley & Sons, New York 1972

[2] Burghardt R., *Freely falling observers*. http://arg.or.at/Wpdf/Wff.pdf

[3] Burghardt R., *Free fall in the Schwarzschild field.* http://arg.or.at/Wpdf/WSsff.pdf

[4] Burghardt R*., Einstein's elevator*. http://arg.or.at/Wpdf/WSskr.pdf

[5] Burghardt R., *Free fall and time function*. http://arg.or.at/Wpdf/WTime.pdf

[6] Burghardt R., *Fall time in the Schwarzschild field.* http://arg.or.at/Wpdf/WMTW.pdf

[7] Burghardt R. *Spacetime curvature*. http://arg.or.at/EMono.htm

[8] Misner, Thorne, and Wheeler, Gravitation, Freeman, San Francisco 1973