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TRANSFORMATIONS IN DE SITTER AND LANCZOS MODELS II.

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Contents

Abstract: In a previous paper [1] we have associated pseudo-rotations or Lorentz transformations to known coordinate transformations for the de Sitter cosmos, the Lanczos cosmos, the Lanczos-like cosmos, and the anti-de Sitter universe. In this paper we will calculate the field quantities and the field equations for these models. We also note that the usual classification of the models with the curvature parameter as done in the FWR models is not reliable.

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1. INTRODUCTION

In the paper (I) we derived pseudo-rotations and Lorentz transformations that mediate between the comoving observer systems (A) and the non-comoving observer systems (B). With these we are able to convert the field quantities of de Sitter family from one system to the other system, without resorting to a coordinate system which is a considerable facilitation. The use of the tetrad method and the associated Ricci-rotation coefficients allows us a better understanding of the geometric background of the models. In particular, it can be elucidated why the FWR curvature parameter k provides inaccurate statements concerning the de Sitter model.

2. PRELIMINARY REMARKS

Before dealing with the individual models, we give a brief overview of the mathematical methods which allow us an insight into the geometric structure of the models. We start with the Ricci-rotation coefficients which we split according to

$$A_{mn}{}^{s} = B_{mn}{}^{s} + C_{mn}{}^{s} + U_{mn}{}^{s} \tag{2.1}$$

Therein is

$$\begin{gathered} B_{mn}{}^{s} = b_m B_n b^s - b_m b_n B^s, \quad C_{mn}{}^{s} = c_m C_n c^s - c_m c_n C^s, \quad U_{mn}{}^{s} = h_m{}^{s} U_n - h_{mn} U^s \\ b_m = \{0,1,0,0\}, \quad c_m = \{0,0,1,0\}, \quad u_m = \{0,0,0,1\} \\ . \end{gathered} \tag{2.2}$$

$h_{mn} = \mathrm{diag}(1,0,0,1)$ is a submatrix of the tetrad metric $g_{mn} = \mathrm{diag}(1,1,1,1)$.

The Ricci

$$R_{mn} = A_{mn}{}^{s}{}_{|s} - A_{n|m} - A_{rm}{}^{s} A_{sn}{}^{r} + A_{mn}{}^{s} A_s, \quad A_n = A_{rn}{}^{r} \tag{2.3}$$

with the quantities takes (2.1), (2.2) the form

$$\begin{aligned} R_{mn} = &-\left[U^{s}{}_{\underset{1}{\|}s} + U^s U_s\right] h_{mn} \\ &-\left[B_{n\underset{2}{\|}m} + B_n B_m\right] - b_n b_m \left[B^{s}{}_{\underset{2}{\|}s} + B^s B_s\right] \\ &-\left[C_{n\underset{3}{\|}m} + C_n C_m\right] - c_n c_m \left[C^{s}{}_{\underset{3}{\|}s} + C^s C_s\right] \\ -\frac{1}{2} R = &\left[U^{s}{}_{\underset{1}{\|}s} + U^s U_s\right] + \left[B^{s}{}_{\underset{2}{\|}s} + B^s B_s\right] + \left[C^{s}{}_{\underset{3}{\|}s} + C^s C_s\right] \end{aligned} \quad . \tag{2.4}$$

The use of the graded derivatives [8]

$$U_{n\underset{1}{\|}m} = U_{n|m}, \quad B_{n\underset{2}{\|}m} = B_{n|m} - U_{mn}{}^{s} B_s, \quad C_{n\underset{3}{\|}m} = C_{n|m} - U_{mn}{}^{s} C_s - B_{mn}{}^{s} C_s \tag{2.5}$$

proves to be advantageous. The structure (2.4) can be applied to spherically symmetric static, expanding or collapsing systems.

The transition from the comoving observer system m' to the non-comoving system m takes place with a pseudo-rotation L, which is at best a Lorentz transformation. The Ricci-rotation coefficients transform inhomogeneously

$$'A_{m'n'}{}^{s'} = A_{m'n'}{}^{s'} + 'L_{m'n'}{}^{s'}, \quad 'L_{m'n'}{}^{s'} = L_s^{s'} L^s_{n'|m'} . \tag{2.6}$$

Since the pseudo-rotation takes place in the [1,4]-slice of the space the inhomogeneous term can be simplified as

$$'L_{m'n'}{}^{s'} = h_{m'}{}^{s'}\,'L_{n'} - h_{m'n'}\,'L^{s'}, \quad 'L_{n'} = 'L_{s'n'}{}^{s'} = \left\{'L_{4'1'}{}^{4'}, 'L_{1'4'}{}^{1'}\right\}. \tag{2.7}$$

Using (2.6) the Ricci transforms according to

$$R_{m'n'} = L^{m\ n}_{m'n'} R_{mn} + L_{m'n'} \ .$$

If the Ricci should be invariant with respect to the transformation the second term in the above relation must disappear. One can show that this term takes for spherically symmetric systems, after a pseudo-rotation in the [1,4]-slice and after some calculations the form

$$'L^{s'}{}_{|s'} + 'U_{s'}\,'L^{s'} = 0, \quad 'U_{s'} = L^s_{s'} U_s + 'L_{s'}, \quad L_s = -L_s^{s'}\,'L_{s'}, \quad L^s{}_{|s} + U_s L^s = 0 . \tag{2.8}$$

Thus, (2.8) is the condition for invariance of the Ricci for the models we treated and the guarantee that the transformations were prepared correctly. With this we have the tools at hand with which we will examine the four classic models.

3. THE MODELS

I. Since *de Sitter* has given his model in a non-comoving coordinate system, we start with this representation, and we execute the transformation of the field quantities into in the comoving system. From the static metric we borrow the 4-bein system

$$\overset{1}{e}_1 = \frac{1}{\cos\eta}, \quad \overset{2}{e}_2 = r, \quad \overset{3}{e}_3 = r\sin\vartheta, \quad \overset{4}{e}_4 = \cos\eta . \tag{3.1}$$

With the quantities

$$a = \cos\eta = \sqrt{1 - r^2/\mathcal{R}_0^2} = 1/\alpha, \quad v = r/\mathcal{R}_0 = \sin\eta \tag{3.2}$$

we calculate the Ricci-rotation coefficients for the non-comoving system. With the reciprocal values of (3.1) we can note the partial derivatives

$$\partial_1 = a\frac{\partial}{\partial r}, \quad \partial_2 = \frac{1}{r}\frac{\partial}{\partial\vartheta}, \quad \partial_3 = \frac{1}{r\sin\vartheta}\frac{\partial}{\partial\varphi}, \quad \partial_4 = \frac{1}{a}\frac{\partial}{i\partial t} \tag{3.3}$$

and we draw from (2.2)

$$B_m = \left\{\frac{a}{r}, 0,0,0\right\}, \quad C_m = \left\{\frac{a}{r}, \frac{1}{r}\cot\vartheta, 0,0\right\}, \quad U_m = \left\{-\alpha v\frac{1}{\mathcal{R}_0}, 0,0,0\right\}. \tag{3.4}$$

The lateral field quantities B and C describe the curvatures of the great circles and parallels of the hypersphere. The quantity $E_1 = -U_1$ is the force acting in all radial directions on any point of the hypersphere. This force will pull apart neighboring points. This suggests a possible interpretation of an expansion of the model.

It is easy to show that with these quantities and with (2.4), (2.5) the Einstein field equations are satisfied. Since the metric in comoving coordinates is (I, 4.8), one can derive with

$$\overset{1'}{e}_{1'} = \mathcal{K}, \quad \overset{2'}{e}_{2'} = r, \quad \overset{3'}{e}_{3'} = r\sin\vartheta, \quad \overset{4'}{e}_{4'} = 1$$
$$\mathcal{K} = e^{\psi'}, \quad r = \mathcal{K}r', \quad dx^{4'} = \mathcal{R}_0 di\psi' = idt' \tag{3.5}$$

the field quantities

$$B_{m'} = \left\{\frac{1}{r}, 0, 0, -\frac{i}{\mathcal{R}_0}\right\}, \quad C_{m'} = \left\{\frac{1}{r}, \frac{1}{r}\cot\vartheta, 0, -\frac{i}{\mathcal{R}_0}\right\}, \quad {}'U_{m'} = \left\{0,0,0,-\frac{i}{\mathcal{R}_0}\right\}. \tag{3.6}$$

We recognize that the numerical identity

$$B_{4'} \overset{*}{=} C_{4'} \overset{*}{=} {}'U_{4'}$$

is valid, ie that the expansion scalar ${}'u^{m'}{}_{||m'}$ is shown correctly. Since the Ricci is form-invariant, ie, the equations (2.4), (2.5) apply equally to the primed system, Einstein's field equations are satisfied with the above field quantities.

Our intention was to deepen the problem of transformations, ie to investigate whether the coordinate transformations not only lead to a mathematical change of the representation, but are accompanied by the possibility of a physical interpretation. We want to check whether the coordinate transformations can be attributed to Lorentz transformations. A Lorentz transformation should transfer the system from a state of motion to another state. We also want to try to derive the quantities (3.6) from the quantities (3.4) with a Lorentz transformation, that is without resorting to the coordinate transformation.

We bring to mind that the Ricci-rotation coefficients (2.1) generally transform inhomogeneously. However, the lateral field quantities behave as tensors under a transformation in the [1,4]-subspace. Therefore, the transformation law (2.6) with the Lorentz term (2.7) is reduced to

$${}'U_{m'n'}{}^{s'} = U_{m'n'}{}^{s'} + {}'L_{m'n'}{}^{s'} \tag{3.7}$$

With $L^1_{1'} = \alpha, L^1_{4'} = -i\alpha v$ one first obtains

$${}'L_{1'} = i\alpha^2 v_{|4'}, \quad {}'L_{4'} = -i\alpha^2 v_{|1'}$$

and from this with (3.2)

$${}'L_{m'} = \left\{\alpha^2 v \frac{1}{\mathcal{R}_0}, 0, 0, -i\alpha^2 \frac{1}{\mathcal{R}_0}\right\}. \tag{3.8}$$

From (2.8) we again obtain (3.6), last equation, and we recognize that the quantiy ${}'U$ does not contain a space-like part. We remember that in the comoving system the coordinate time coincides with the proper time, that means that no force can be derived from the time-like part of the line element.

The subequations of Einstein's field equations in the comoving system are

$$'U^{s'}{}_{\underset{1}{||}s'} + 'U^{s'}\,'U_{s'} = -\frac{1}{\mathcal{R}_0^2}$$

$$B_{m'\underset{2}{||}n'} + B_{m'}B_{n'} = -h_{m'n'}\frac{1}{\mathcal{R}_0^2}, \qquad B^{s'}{}_{\underset{2}{||}s'} + B^{s'}B_{s'} = -\frac{2}{\mathcal{R}_0^2} \tag{3.9}$$

$$C_{m'\underset{3}{||}n'} + C_{m'}C_{n'} = -\left(h_{m'n'} + b_{m'}b_{n'}\right)\frac{1}{\mathcal{R}_0^2}, \qquad C^{s'}{}_{\underset{3}{||}s'} + C^{s'}C_{s'} = -\frac{3}{\mathcal{R}_0^2}$$

and provide together the Einstein tensor

$$G_{m'n'} = -g_{m'n'}\frac{3}{\mathcal{R}_o^2}. \tag{3.10}$$

The invariance of the Ricci addressed in (2.8) is satisfied.

From the stress-energy tensor

$$\kappa T_{m'n'} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}\frac{3}{\mathcal{R}_0^2}, \quad \kappa p = -\frac{3}{\mathcal{R}_0^2}, \quad \kappa\mu_0 = \frac{3}{\mathcal{R}_0^2} \tag{3.11}$$

one can read the expressions for the pressure and density of matter. Since is $\mathcal{R}_0$ a constant, both the pressure and matter density are constant. Both quantities are usually expressed by the cosmological constant $\lambda = 3/\mathcal{R}_0^2$. The latter interpretation has the advantage that one does not have to do with an expanding universe in which the pressure and density of matter are independent of time. The dS model has another peculiarity. The field quantities in the static system (3.4) satisfy the field equations (3.9), if one writes them down in the unprimed form. Since, according to (3.11) the equation of state

$$p + \mu_0 = 0 \tag{3.12}$$

applies for the dS-cosmos, the stress-energy tensor in the non-comoving system

$$T_{mn} = -g_{mn}p + (p + \mu_0)'u_m\,'u_m \tag{3.13}$$

is reduced to the form listed in (3.11). It has the same components in the comoving and non-comoving systems. The observer at rest cannot locate any matter current caused by the expansion. This problem has been pointed out by Mitra [2]. The dS-cosmos cannot meet our expectations regarding a useful physical model.

The dS model has still another peculiarity. From (3.6) we take the spatial components of the field quantities of the comoving system and we compare them with those of the non-comoving system (3.4)

$$B_{\alpha'} = \left\{\frac{1}{r},0,0\right\}, \quad C_{\alpha'} = \left\{\frac{1}{r},\frac{1}{r}\cot\vartheta,0\right\}, \quad 'U_{\alpha'} = \{0,0,0\}, \tag{3.14}$$

$$B_{\alpha} = \left\{\frac{a}{r},0,0\right\}, \quad C_{\alpha} = \left\{\frac{a}{r},\frac{1}{r}\cot\vartheta,0\right\}, \quad U_{\alpha} = \left\{-\alpha v\frac{1}{\mathcal{R}_0},0,0\right\}. \tag{3.15}$$

It can be recognized that (3.14) is consistent with the field quantities of a spatially flat geometry. One gets the impression that a Lorentz transformation could make a flat geometry from a curved one. Also, according to the FWR-method the curvature parameter $k = 0$ is associated to the dS-cosmos. This classifies the 3-dimensional space as flat. We believe that a transformation of the reference system cannot change the spatial structure, but only the perspective of an observer to this structure. We want to get to the bottom of the matter.

In (I.4.6) we have geometrically explained the relative velocity and the Lorentz factor

$$v = \sin\eta = \frac{r}{\mathcal{R}_0}, \quad \alpha = \frac{1}{\cos\eta} = \frac{1}{\sqrt{1-r^2/\mathcal{R}_0^2}} = \frac{1}{a} \tag{3.16}$$

and thus constituted the matrix of the Lorentz transformation. The latter can also be explained with the Lorentz angle χ. Thus, we can put

$$L^{m}_{m'} = \begin{pmatrix} \alpha & & & -i\alpha v \\ & 1 & & \\ & & 1 & \\ i\alpha v & & & \alpha \end{pmatrix} = \begin{pmatrix} \cos i\chi & & & \sin i\chi \\ & 1 & & \\ & & 1 & \\ -\sin i\chi & & & \cos i\chi \end{pmatrix}. \tag{3.17}$$

Hence the relation

$$\cos\eta \cos i\chi = 1 \tag{3.18}$$

holds. This means that the angle η in the [0,1]-plane of the embedding space is assigned to the angle χ in the [1', 4']-plane of the physical space. If we write

$$B_{\alpha} = \left\{ \frac{\cos\eta}{r}, 0, 0 \right\} \tag{3.19}$$

we have after the Lorentz transformation and with (3.18)

$$B_{1'} = L^{1}_{1'} B_1 = \cos i\chi \frac{1}{r} \cos\eta = \frac{1}{r}, \quad B_{4'} = L^{1}_{4'} B_1 = -\sin i\chi \frac{1}{r} \cos\eta = -i\alpha v a \frac{1}{r} = -\frac{i}{\mathcal{R}_0}$$

and thus

$$B_{m'} = \left\{ \frac{1}{r}, 0, 0, -\frac{i}{\mathcal{R}_0} \right\} = \left\{ \cos i\chi \frac{1}{r} \cos\eta, 0, 0, -\sin i\chi \frac{1}{r} \cos\eta \right\}. \tag{3.20}$$

The second brackets still contain the information about the curvature of space, but according to (3.18) this information is hidden to the comoving observer. Thus, it is evident that the FWR-classification with $k = 0$ is not reliable concerning the dS-cosmos.

It should also be noted that the problem is similar to the one Lemaître has described for the Schwarzschild theory. For the free fall Lemaître has specified a comoving coordinate system. This can be assigned to an observer system which is associated to a freely falling observer. In this system the lateral field quantities appear to be flat, they have the structure (3.6). This corresponds to the principle of Einstein's elevator. If the observers follow the forces of the field, they experience no accelerations. This allows another interpretation of the dS-model: A static cosmos is equipped at each point, with a force field that points away from this point into all directions of space. This is valid for any point, in the preceding formulae the position of $\eta' = 0$ was chosen arbitrarily. Observers who follow these forces do not experience a radial acceleration. This effect must not be interpreted as an expansion of space, but as a motion of 'freely falling' observers. Based on these considerations the dS-cosmos does not even approximately seem to be an image of our universe.

II. For the *model of Lanczos* we note the 4-bein system

$$\overset{1'}{e}_{1'} = \frac{\mathrm{ch}\psi'}{\cos\eta'}, \quad \overset{2'}{e}_{2'} = r, \quad \overset{3'}{e}_{3'} = r\sin\vartheta, \quad \overset{4'}{e}_{4'} = 1,$$
$$\overset{1}{e}_{1} = \frac{1}{\cos\eta}, \quad \overset{2}{e}_{2'} = r, \quad \overset{3}{e}_{3} = r\sin\vartheta, \quad \overset{4}{e}_{4} = \cos\eta, \tag{3.21}$$
$$\mathcal{K} = \mathrm{ch}\psi', \quad r = \mathcal{K} r'.$$

With this we calculate the components of the Ricci-rotation coefficients

$$B_m = \left\{\frac{1}{r}\cos\eta, 0,0,0\right\}, \quad C_m = \left\{\frac{1}{r}\cos\eta, \frac{1}{r}\cot\vartheta, 0,0\right\}, \quad U_m = \left\{-\frac{1}{\mathcal{R}_0}\tan\eta, 0,0,0\right\},$$
$$B_{m'} = \left\{\frac{1}{r}\cos\eta', 0,0,-\frac{i}{\mathcal{R}_0}\mathrm{th}\psi'\right\}, \quad C_{m'} = \left\{\frac{1}{r}\cos\eta', \frac{1}{r}\cot\vartheta, 0, -\frac{i}{\mathcal{R}_0}\mathrm{th}\psi'\right\}, \tag{3.22}$$
$$'U_{m'} = \left\{0,0,0,-\frac{i}{\mathcal{R}_0}\mathrm{th}\psi'\right\}.$$

With $r = \mathcal{R}_0 \sin\eta, \mathcal{R} = \mathcal{R}_0 \mathrm{ch}\psi'$ and $\sin\eta = \sin\eta' \mathrm{ch}\psi'$ we can write $B_1 = \frac{1}{\mathcal{R}_0}\cot\eta$ and $B_{1'} = \frac{1}{\mathcal{R}}\cot\eta'$. Also the Lanczos cosmos contains forces directed away from any point. The comoving observer is no longer exposed to these forces. The tidal forces $B_{4'}, C_{4'}, 'U_{4'}$ act on it.

To convert these quantities into one another, one needs the Lorentz transformation from (I.5.5). With the relative velocity of (I.6.4) we can calculate

$$'L_{m'} = \left\{\frac{\cos\eta'}{\cos\eta}\tan\eta\frac{1}{\mathcal{R}_0}, 0,0,-\frac{1}{\cos\eta}\mathrm{th}\psi'\frac{i}{\mathcal{R}_0}\right\}. \tag{3.23}$$

The field quantities (3.22) transform correctly according to

$$B_{m'} = 'L^m_{m'} B_m, \quad C_{m'} = 'L^m_{m'} C_m, \quad 'U_{m'} = 'L^m_{m'} U_m + 'L_{m'}. \tag{3.24}$$

With (2.4) they give a stress-energy tensor as (3.11) which provides the same difficulties as that of the dS-cosmos. The Ricci is Lorentz invariant as expected.

III. The structure of the *Lanczos-like cosmos* is even less close to the universe we are living in than the ones discussed earlier. Nevertheless, we will find that the quantities derived for this cosmos satisfy the field equations. After all, Einstein's field equations are nonlinear differential equations of 2^{nd} order, the solutions for which no reference need be made to their physical usability. The 4-bein systems have the components

$$\overset{1'}{e}_{1'} = \frac{\mathrm{sh}\psi'}{\cos\eta'}, \quad \overset{2'}{e}_{2'} = r, \quad \overset{3'}{e}_{3'} = r\sin\vartheta, \quad \overset{4'}{e}_{4'} = 1,$$
$$\overset{1}{e}_{1} = \frac{1}{\sqrt{1-\mathrm{sh}^2\eta}}, \quad \overset{2}{e}_{2'} = r, \quad \overset{3}{e}_{3} = r\sin\vartheta, \quad \overset{4}{e}_{4} = \sqrt{1-\mathrm{sh}^2\eta}, \tag{3.25}$$
$$\mathcal{K} = \mathrm{sh}\psi', \quad r = \mathcal{K} r'.$$

Thus, for the components of the Ricci-rotation coefficients one has

$$B_m=\left\{\frac{1}{r}\sqrt{1-sh^2\eta},0,0,0\right\},\quad C_m=\left\{\frac{1}{r}\sqrt{1-sh^2\eta},\,\frac{1}{r}\cot\vartheta,0,0\right\},$$

$$U_m=\left\{-\frac{1}{\mathcal{R}_0}\frac{sh\eta}{\sqrt{1-sh^2\eta}},0,0,0\right\},$$

$$B_{m'}=\left\{\frac{1}{r}\cos\eta',0,0,-\frac{i}{\mathcal{R}_0}cth\psi'\right\},\quad C_{m'}=\left\{\frac{1}{r}\cos\eta',\,\frac{1}{r}\cot\vartheta,0,-\frac{i}{\mathcal{R}_0}cth\psi'\right\}, \qquad (3.26)$$

$$'U_{m'}=\left\{0,0,0,-\frac{i}{\mathcal{R}_0}cth\psi'\right\}.$$

With the Lorentz transformation from (I.5.8) and

$$'L_{m'}=\frac{1}{1-sh^2\eta}\left\{\frac{1}{\mathcal{R}_0}sh\eta\, ch\eta',0,0,-\frac{i}{\mathcal{R}_0}cth\psi'\right\} \qquad (3.27)$$

the field quantities (3.26) transform with each other according to (3.24) and satisfy the Einstein field equations and the invariance of the Ricci.

IV. For the *anti-de Sitter cosmos* the basic relations

$$\overset{1'}{e}_{1'}=\frac{\sin\psi'}{ch\eta'},\quad \overset{2'}{e}_{2'}=r,\quad \overset{3'}{e}_{3'}=r\sin\vartheta,\quad \overset{4'}{e}_{4'}=1,$$

$$\overset{1}{e}_{1}=\frac{1}{ch\eta},\quad \overset{2}{e}_{2'}=r,\quad \overset{3}{e}_{3}=r\sin\vartheta,\quad \overset{4}{e}_{4}=ch\eta, \qquad (3.28)$$

$$\mathcal{K}=\sin\psi',\quad r=\mathcal{K}r'$$

are valid. Therewith one calculates the field quantities

$$B_m=\left\{\frac{1}{r}ch\eta,0,0,0\right\},\quad C_m=\left\{\frac{1}{r}ch\eta,\,\frac{1}{r}\cot\vartheta,0,0\right\},$$

$$U_m=\left\{\frac{1}{\mathcal{R}_0}th\eta,0,0,0\right\},$$

$$B_{m'}=\left\{\frac{1}{r}ch\eta',0,0,-\frac{i}{\mathcal{R}_0}\cot\psi'\right\},\quad C_{m'}=\left\{\frac{1}{r}ch\eta',\,\frac{1}{r}\cot\vartheta,0,-\frac{i}{\mathcal{R}_0}\cot\psi'\right\}, \qquad (3.29)$$

$$'U_{m'}=\left\{0,0,0,-\frac{i}{\mathcal{R}_0}\cot\psi'\right\}.$$

With

$$'L_{m'}=-\left\{\frac{1}{\mathcal{R}_0}\frac{ch'}{ch\eta}th\eta,0,0\frac{i}{ch^2\eta}cth\psi'\right\} \qquad (3.30)$$

one establishes the connection of the quantities (3.29). With these expressions the Einstein field equations are satisfied. However, pressure and mass density have the sign opposite to the corresponding quantities of the de Sitter universe. This result and also the periodic lapse of time pose the AdS cosmos far away from reality.

4. CONCLUSIONS

In a previous paper we have studied the four models of the dS family. We have assigned Lorentz transformations to the transformations between comoving and non-comoving coordinates, transformations which are already known in the literature. These Lorentz transformations establish a relation between comoving and non-comoving *observer systems.* In the present paper, we have decomposed the Ricci-rotation coefficients into its components with respect to these observer systems. We have shown that in all considered models which are presented in so-called static systems, forces act in any point of the universe into the radial directions. This indicates a possible expansion of the universe. If an observer follows these forces, he is not exposed to acceleration, however, tidal forces act on him. One could say that the observer follows the expansion of the universe. However, against such an interpretation runs that by the observer at rest no matter transport can be experienced.

5. LITERATURE

[1] Burghardt R., *Transformations in de Sitter and Lanczos models I.* arg.or.at/Wpdf/Wtrans1.pdf

[2] Mitra A., *When can an "Expanding Universe" look "Static" and vice versa: A comprehensive study.* Int. Journ. Mod. Phys. **24**, 1550032, 2015