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THE RELATIVITY OF ACCELERATION

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Contents

Abstract. The validity of the relativity of acceleration is investigated in view of Treder's theory of tetrads. Employing the methods of field theory, we analyze the effects of Newton's bucket and Sagnac's experiment. Moreover, the paper proves that the results yielded by the experiments mentioned, do not contradict the basic ideas of the relativity of acceleration. The principle of constancy of light velocity is also valid in general relativity.

German title: Zur Relativität der Beschleunigung

Inhaltsübersicht. Die Relativität der Beschleunigung wird im Lichte der Trederschen Tetradentheorie diskutiert. Die Erscheinungen des Newtonschen Eimerversuchs und Sagnac-Experiments werden feldtheoretisch untersucht und gezeigt, daß beide Experimente nicht im Widerspruch zur Relativität der Beschleunigung stehen. Das Prinzip der Konstanz der Lichtgeschwindigkeit behält auch im Rahmen der allgemeinen Relativitätstheorie seine Gültigkeit.

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1. INTRODUCTION

The relativization of the velocity concept and its consequences were profound changes which the theory of special relativity has caused in the field of physics. A further generalization in the sense of relativism would call in question the measurability of absolute accelerations. In the conventional formulation of general relativity, such a principle is not recognized. Foremost H.-J. Treder (1966/67) has shown that with a careful design of measurement specifications, the relativity of acceleration can be expressed in the context of general relativity. Treder's concept is mathematically reflected in the use of tetrads which constitute rods and clocks.

Essentially two observations seem to contradict the principle of relativity generalized in such a way: Newton's bucket experiment and the Sagnac experiment.

In the rotating bucket there occurs a peripheral raise of the water level, caused by the centrifugal forces which originated in the rotation of the bucket. Thus, the rotational motion should be an absolute motion, because such an effect cannot be observed in non-rotating systems. A relativization of the path acceleration occurring during the rotation and the forces involved would imply that the same forces also occur if the bucket is at rest and the surrounding space is rotating around it. H. Thirring (1918a, b, 1921) has shown that the rotating fixed stars can produce centrifugal and Coriolis forces. These are certainly too weak to account for those phenomena in the bucket. However, a representation of the Einstein field equations in the Lorentz space (D.-E. Liebscher, H.-J. Treder 1970, R. Burghardt 1979, 1980) shows that non-linear field terms are present in these equations. These field terms are physically to be interpreted as gravitational energy and energy flow and are equivalent to the rotating masses of the fixed stars. Those energies exert forces of the required strength on then system located around the center of rotation. (In the mathematical analysis in Sec. 2 we will completely leave aside the fixed stars.) In the Lorentz space the field equations of gravitation take a Maxwell-like form, and the physical principles which are described by them also recall the theory of electromagnetism: The gravitational energy corresponds to a negative field mass (F. Hund 1947) which is repulsive and causes centrifugal forces, the flow of gravitational energy (Poynting vector) generates the Coriolis dipole field. The nonlinearity of the Einstein field equations is responsible for the field mechanism with back reaction and also leads to an extension of Mach's principle (H. Hönl, H. Dehnen 1966): The distribution of energy and momentum density of matter and the gravitational field alone suffices to determine the inertial forces in any reference system.

The optical experiment of Sagnac (1913), the equivalent of the Michelson experiment is seemingly also a strong argument for the absoluteness of rotation: light beams are circling in opposite directions on a plate with a mirror system which is adjusted in such a way that the rays meet after a circulation and cause interferences. If the plate is rotated the interference fringes are shifted in dependence of the angular velocity. The cause is believed to be the following: the light has different velocities in opposite directions (M. P. Langevin 1921, 1937, E. J. Post 1967), thus indicating the absoluteness of the rotational movement. Only in the system at rest both light beams have the same velocity.

Detached from this special experimental arrangement one could also argue that a uniformly moving observer that deviates temporarily from its straight path, changes permanently his philosophy: The principle of constancy of the velocity of light is only valid for the straight parts of the route, any deviation from it leads to a violation of the principle of constancy. This possible consequence of the Sagnac experiment is not only unsatisfactory, but is also contrary to the principles of general relativity. Therefore we look for another interpretation of the undoubtedly observable fringe shift on the Sagnac interferometer: The experimental arrangement can be regarded as being relatively at rest, but it is orbited by gravitational energy. The forces arising from this energy lead to elongations and contractions of the optical paths and to changes in the physical time flow. Opposite beams take differing paths being responsible for the measured fringe shift. But for this they need times differently in length. The quotients of these paths and times, the velocities of light, are equal. Therefore the principle of constancy for accelerated reference frames applies and the relativity of rotational motion is ensured. In Sec. 2 and 3 we will show that it is the very gravitational forces that affect the Newtonian bucket and the light paths in the Sagnac experiment.

In Sec. 4 the field equations for a rotating system will be discussed, in Sec. 5 we will take a brief look at a rotator with differential rotation law.

2. ROTATING SYSTEMS

To illuminate mathematically the mechanisms indicated in the previous Section, it would be useful to investigate a rotating solution of Einstein's field equations. We avoid the diversity of the known rotating models by taking the simplest system at hand: a family of rotating observers in the Minkowski space. Since the central mass is missing the Riemann curvature tensor vanishes. The model cannot be attributed either to the special theory of relativity, because it allows non inertial observer systems, or to the general theory of relativity, because the gravitational field is closely linked to the space-time curvature. The flat problem is merely the field theoretical formulation of the classical rotation problem, but it shows the essential structures of a rotating gravitation model. Because of its simplicity the effects of the forces on an experimental assembly can be described without special mathematical effort. Genuine gravity models can be reduced to this system. Thus, this simplification is justified.

Our investigations are based on the Euclidean line element in spherical coordinates:

$$ds^2 = dr^2 + r^2 d\vartheta^2 + r^2 \sin^2\vartheta d\varphi^2 + \left(dx^4\right)^2, \quad dx^4 = idt,\ c = 1. \tag{2.1}$$

After a transformation into a rotating coordinate system with

$$\varphi \to \varphi + \omega t \tag{2.2}$$

the line element has the form

$$ds^2 = dr^2 + r^2 d\vartheta^2 + \sigma^2 d\varphi^2 - 2i\omega\sigma^2 d\varphi dx^4 + \alpha^{-2}\left(dx^4\right)^2. \tag{2.3}$$

Here

$$\sigma = r\sin\vartheta \tag{2.4}$$

is the radius of the parallels of latitude with respect to the equatorial plane and

$$\alpha = 1\Big/\sqrt{1-\omega^2\sigma^2} \tag{2.5}$$

the Lorentz factor of a relative motion, still to be discussed. In the literature, the metric (2.1) is often associated with a physical system at rest, (2.3) however, with a rotating model. We want to notice that in both cases it is the same metric in two different representations. A coordinate transformation (which is in the present case a Galilean), only changes the description, but does not alter the physical situation. This problem is discussed in the appendix in more detail. The rotational motion which we want to analyze is not linked to the metric, but to an observer system $(1=r,\ 2=\vartheta,\ 3=\varphi,\ 4=it)$

$$(A)\qquad \begin{matrix} \overset{1}{e}_1=1, & \overset{2}{e}_2=r, & \overset{3}{e}_3=\alpha\sigma, & \overset{4}{e}_3=-i\alpha\omega\sigma^2, & \overset{4}{e}_4=1/\alpha \\ \underset{1}{e}^1=1, & \underset{2}{e}^2=1/r, & \underset{3}{e}^3=1/\alpha\sigma, & \underset{3}{e}^4=\ i\alpha\omega\sigma, & \underset{4}{e}^4=\alpha \end{matrix} \tag{2.6}$$

which arises from the system

$$(B)\qquad \begin{matrix} \overset{1}{e}_1=1, & \overset{2}{e}_2=r, & \overset{3}{e}_3=\sigma, & \overset{3}{e}_4=-i\omega\sigma, & \overset{4}{e}_4=1 \\ \underset{1}{e}^1=1, & \underset{2}{e}^2=1/r, & \underset{3}{e}^3=1/\sigma, & \underset{3}{e}^3=\ i\omega, & \underset{4}{e}^4=1 \end{matrix} \tag{2.7}$$

being global at rest and generated by a generalized Lorentz transformation $\{i\alpha\omega\sigma,\alpha\}$ with the relative velocity $\omega\sigma$ dependent on σ and with the Lorentz factor α. The system A is not inertial, forces in it are measurable which we calculate using a methodology specified earlier (Burghardt 1979, 1980): The three-rank quantity A has with respect to the Lorentz space the components $(\alpha = 1,2,3)$:

$$\begin{matrix} A_{2\alpha}{}^2 = B_\alpha, \quad A_{3\alpha}{}^3 = S_\alpha + F_\alpha, \quad A_{4\alpha}{}^4 = -E_\alpha \\ A_{\alpha\beta}{}^4 = H_{\alpha\beta}, \quad A_{4\beta}{}^\gamma = H^\gamma{}_\beta, \quad A_{\alpha 4}{}^\gamma = H^\gamma{}_\alpha \end{matrix}, \tag{2.8}$$

wherin

$$B_\alpha = \delta^1_\alpha S_1, \quad S_\alpha = (\ln\sigma)_{|\alpha}, \quad \sigma_\alpha = \sigma_{|\alpha} = \{\sin\vartheta, \cos\vartheta, 0\} \tag{2.9}$$

are geometric quantities and

$$E_\alpha = \alpha^2\omega^2\sigma\sigma_\alpha, \quad H_{\alpha 3} = i\alpha^2\omega\sigma_\alpha \tag{2.10}$$

are the relativistic generalizations of the centrifugal and Coriolis forces. The centrifugal force is perpendicular to the rotation axis, the Coriolis force parallel to the axis, as is obvious by the dual form

$$H_1 = i\alpha^2\omega\cos\vartheta, \quad H_2 = -i\alpha^2\omega\sin\vartheta. \tag{2.11}$$

The quantities A are the coefficients of an observer invariant and coordinate independent transport law, which specifies a parallelism with regard to the 4-bein system $(m = 1, 2, ..., 4)$:

$$v_{m\,||n} = v_{m|n} - A_{nm}{}^s v_s\,. \tag{2.12}$$

The spatial part $(\alpha = 1, 2, 3)$

$$v_{\alpha\wedge\beta} = v_{\alpha|\beta} - A_{\beta\alpha}{}^\gamma v_\gamma \tag{2.13}$$

is invariant with respect to three-dimensional observer transformations and is reduced in the non-relativistic case to the ordinary differentiation law in spherical coordinates.

The equation of motion

$$v^n v_{m\|n} = 0, \quad v_m = 1\Big/\sqrt{1-v^2}\,\{-i v_\alpha, 1\} \tag{2.14}$$

with v as the particle velocity splits after spacetime decomposition into the momentum and energy conservation law

$$\begin{aligned}(\mathbf{v}\ \mathrm{grad})\mathbf{p} + \dot{\mathbf{p}} &= \mu(\mathbf{E} + 2\mathbf{v}\times\mathbf{H}),\\ (\mathbf{v}\ \mathrm{grad})\mu + \dot{\mu} &= \mu(\mathbf{E}\mathbf{v}),\end{aligned} \tag{2.15}$$

where μ_0 is the rest mass of the particle and the correlations

$$v^\alpha \mathrel{\hat{=}} \mathbf{v}, \quad E^\alpha \mathrel{\hat{=}} \mathbf{E}, \quad H^{\alpha\beta} \mathrel{\hat{=}} i\mathbf{H}, \quad p^\alpha \mathrel{\hat{=}} \mu_0\,\mathbf{v}\Big/\sqrt{1-v^2}, \quad \mu = \mu_0\Big/\sqrt{1-v^2} \tag{2.16}$$

are used. The operator grad is explained by (2.13). The right side of the equations (2.15) is the Lorentz force density of the field and is responsible for the deviations from straightness of the particle path.

3. THE EFFECT OF THE FORCES

It is immediately apparent that the forces (2.10) are responsible for the phenomena on the Newtonian bucket. They were derived from the global rotating system A. Applying the same mathematical procedure to the global system B at rest no field strengths arise. Both systems are connected by the generalized Lorentz transformation

$$A_3^{3'} = A_4^{4'} = \alpha, \quad A_4^{3'} = -A_3^{4'} = i\alpha\omega\sigma. \tag{3.1}$$

(The indices refer to the Lorentz space.) The field strengths can be converted with the inhomogeneous transformation law

$$A_{n'm'}{}^{s'} = A_{n'm's}^{n\,m\,s'} A_{nm}{}^{s} + A_s^{s'} A^s_{m'|n'} \tag{3.2}$$

mediating between the systems. The physical components in the system B vanish completely. We have referred to A as globally rotating and to B as globally at rest. However, this is a simplified way of expression related to classical mechanics. If our views on the relativity of the acceleration should survive, the terms rotating and being at rest must be interchangeable. To start with, it is only stated by (3.1) that the motions of the systems are relative to each other, whereby the relative velocity is not a constant of the transformation, but a vector which permanently changes its direction. (The angular velocity ω is the third component of a Lorentz vector ω_α.) Considering A to be at rest and B to rotate, the expressions on the right side of (2.15) no longer have the classical meaning of pseudo-forces (Scheinkräfte), but act as external forces on the system at rest, caused by mechanisms of the rotating space. In the next Section this we will discuss in more detail.

For the interpretation of Sagnac's experiment we must make use of the nonholonomicity of the observer system A. The non-commutativity of the partial derivatives in the Lorentz space results in the relation

$$\Phi_{|[mn]} = S_{mn}{}^{r}\Phi_{|r}, \quad S_{mn}{}^{r} = -A_{[mn]}{}^{r}. \tag{3.3}$$

The Lorentz components of the position vector are not integrable:

$$\Delta x^r = \iint d \wedge dx^r = 2\iint S_{mn}{}^r dx^m \wedge dx^n . \tag{3.4}$$

It is sufficient to specify the components of S to a first approximation, whereby we temporarily abandon the natural unit system and replace ω with ω/c. Then the Lorentz factor is $\alpha \approx 1$ and

$$2S_{13}{}^3 = S_1 + F_1 \approx \frac{1}{r}, \quad 2S_{14}{}^3 = 0, \quad 2S_{14}{}^4 = -E_1, \quad 2S_{13}{}^4 = -2H_{13} \approx -2i\omega/c . \tag{3.5}$$

Following the path around the Sagnac disc in the direction of rotation, one has

$$\Delta x_1^3 = 2\iint \left[S_{13}{}^3 dx^1 dx^3 + S_{14}{}^3 dx^1 dx^4 \right].$$

Going back to the system at rest and finally using for integration the holonomic coordinates (i) of the Einstein space one obtains

$$\Delta x_1^3 = 2\pi r - \omega r t$$

and for the path in the opposite direction of the rotation

$$\Delta x_2^3 = 2\pi r + \omega r t .$$

The difference between the paths with different lengths is

$$\Delta s = -2\omega r t . \tag{3.6}$$

In the course of time

$$\Delta x_1^4 = 2\iint \left[S_{13}{}^4 dx^1 dx^3 + S_{14}{}^4 dx^1 dx^4 \right] = -2i\iint \frac{\omega r}{c} dr \left(d\varphi - \omega t \right)$$

different intervals arise depending on the sense of rotation (A is the circulated area)

$$\Delta t_1 = -\frac{\omega}{c^2}\int r^2 d\varphi = -2\frac{\omega A}{c^2}, \quad \Delta t_2 = 2\frac{\omega A}{c^2},$$

if one returns to the starting point and performs a time comparison with a residual clock, so that one has

$$\Delta t = \frac{4\omega A}{c^2} \tag{3.7}$$

in compliance to J. F. Corum (1977). But we do not follow Corum, who reads from (3.7) a rotation-related change in the frequency of light. Frequency distortions would lead to disintegration of the fringe pattern on the interferometer.

A light beam of the wavelength λ and the oscillation period T covers the path $n\lambda$ on the platform at rest and needs the time $nT = t$, so that

$$n\lambda = cnT = ct . \tag{3.8}$$

On the rotating plate

$$\Delta n\lambda = c\Delta t$$

applies to the two counter-rotating beams with constant c, and if one takes advantage of (3.7) Sagnac's formula for the phase shift reads as

$$\Delta n = \frac{4\omega A}{\lambda c} . \tag{3.9}$$

It has no effect on the experiment whether we consider the platform as rotating or at rest. In both cases, it is the forces (3.5) that result from a relative acceleration and cause the fringe shift. The principle of constancy of the velocity of light is fully applicable. From the metric (2.3) one takes for $ds^2 = 0$

$$\left[\overset{3}{e}_3 dx^3\right]^2 + \left[\overset{4}{e}_3 dx^3 + \overset{4}{e}_4 dx^4\right]^2 = 0$$

while maintaining the system A. If one defines the velocity term stricly in the Lorentz space, then the coordinate invariant relation

$$\frac{dx^{\hat{3}}}{dx^{\hat{4}}} = \pm i, \quad \frac{dx^{\hat{3}}}{d\tau} = \pm c \tag{3.10}$$

applies to the circulating light beam with

$$dx^m = \overset{m}{e}_i dx^i$$

in accordance with the principle of constancy.

4. FIELD EQUATIONS AND CONSERVATION LAWS

The model that we treat as a substitute for rotating gravitational systems is flat. Therefore the relation

$$R_{smnr} = 0 \tag{4.1}$$

is valid. However, if we continue the gravitation-like processing of the model, we have to investigate the vacuum field equations

$$R_{mn} = 0, \quad G_{mn} = R_{mn} - \frac{1}{2} g_{mn} R = 0 . \tag{4.2}$$

Spacetime splitting provides

$$\begin{aligned} G_{\alpha\beta} &= {}'G_{\alpha\beta} + \kappa t_{\alpha\beta} = 0, \\ R_{\alpha 4} &= H^{\beta}{}_{\alpha\wedge\beta} + \kappa t_{\alpha 4} = 0, \\ R_{44} &= E^{\beta}{}_{\wedge\beta} + \kappa t_{44} = 0. \end{aligned} \tag{4.3}$$

${}'G_{\alpha\beta}$ is the mere spatial Einstein tensor. The stress tensor, the Poynting vector, and the energy density of the field are given by

$$\begin{gathered} \kappa t_{\alpha\beta} = \left[E_{\alpha\wedge\beta} - g_{\alpha\beta} E^{\gamma}{}_{\wedge\gamma}\right] - \left[E_{\alpha} E_{\beta} - g_{\alpha\beta} E^2\right] - 2\left[H_{\alpha\gamma} H_{\beta}{}^{\gamma} - \frac{1}{4} g_{\alpha\beta} H^2\right], \\ \kappa t_{\alpha 4} = 2 H_{\alpha\beta} E^{\beta}, \quad \kappa t_{44} = H^2 - E^2, \quad H^2 = H_{\alpha\beta} H^{\alpha\beta}, \quad E^2 = E_{\alpha} E^{\alpha}. \end{gathered} \tag{4.4}$$

From

$$R_{[smn]r} = 0 \tag{4.5}$$

the relations

$$E_{[\alpha\wedge\beta]} = 0, \quad H_{[\alpha\beta\wedge\gamma]} = 0 \tag{4.6}$$

follow. The field equations can quite clearly be represented in symbolic form:

$$\begin{aligned} \mathrm{rot}\mathbf{H} + 2\mathbf{H}\times\mathbf{E} = 0, \quad & \mathrm{div}\,\mathbf{H} = 0, \\ \mathrm{div}\mathbf{E} + \mathbf{H}^2 - \mathbf{E}^2 = 0, \quad & \mathrm{rot}\,\mathbf{E} = 0. \end{aligned} \tag{4.7}$$

In addition, the coordinate invariant differential conservation laws apply:

$$t^{\alpha\beta}{}_{\wedge\beta} = 0, \quad \mathrm{div}(\mathbf{H}\times\mathbf{E}) = 0, \quad \frac{\partial}{\partial t}(H^2 - E^2) = 0. \tag{4.8}$$

Field stresses, momentum, and energy can be localized. The structures (4.7), (4.8) also apply to genuine gravity models. The field mechanisms can easiest be discussed in the Maxwell form: The field energy

$$H^2 - E^2 = -\alpha^4\omega^2\left(2 + \omega^2\sigma^2\right)$$

is negative and repulsive. It is the source of the radial (irrotational) lines of the centrifugal acceleration E. The Coriolis field with the circular (divergence-free) H-lines is generated by the circulation of the field energy. The system itself on which the fields act will be considered at rest. It is the very structures (4.7) which encourage us to believe that the general theory of relativity can be understood as the relativity theory of the acceleration concept.

5. DIFFERENTIAL ROTATION LAW

Although Eqs. (4.7) are transpositions of relativistic mechanics, they cannot be correct. First, they do not represent a genuine gravitational model as a consequence of the additional condition (4.1) and they also contain the restriction $\omega = \mathrm{const.}$, so that the orbital speed $\omega\sigma$ reaches at $\sigma = 1/\omega$ the velocity of light. For $\sigma \to \infty$ the centrifugal and Coriolis forces take infinite values. The model had to be "cut off" at $\sigma = 1/\omega$ at least. To avoid such a radical strategy, some authors (Franklin 1922, Hill 1946, Trocheris 1949) tried to improve the situation with a differential rotation law. They put

$$\omega = \omega(\sigma), \tag{5.1}$$

wherein the angular velocity decreases with increasing axial distance. With a suitable choice of the function (5.1) the orbital velocity reaches the velocity of light only at infinity. Rotating gravity models partly include also the condition $\omega = \mathrm{const.}$ and must be "cut off". Therefore their physical usefulness is questioned. Other models like the solution of Kerr (1963) used the law (5.1), whereby the orbital speed approaches zero at infinity. As a consequence also the centrifugal and Coriolis forces vanish at infinity. This corresponds to the nature of a field theory. Putting the mass parameter of the Kerr model $M = 0$ one obtains a flat model with the differential rotation law

$$\omega = \frac{a}{A}, \quad A^2 = r^2 + a^2, \tag{5.2}$$

wherein a is the linear eccentricity and (A, A, r) are the main axes of a family of ellipsoids of revolution. These represent the equipotential surfaces and coordinate hypersurfaces of an elliptic coordinate system. The use of elliptic coordinates (in the Kerr case these are the general Boyer-Lindquist coordinates) is almost mandatory if one wants to perform further calculations. The constant a is at the same time a parameter for the angular velocity. The line element in elliptical coordinates (Krasinski, 1978)

$$ds^2 = \left(\rho^2/A^2\right)dr^2 + \rho^2 d\vartheta^2 + \sigma^2 d\varphi^2 + \left(dx^4\right)^2, \tag{5.3}$$

$$\rho^2 = r^2 + a^2\cos^2\vartheta, \quad \sigma = A\sin\vartheta \tag{5.4}$$

is obtained by a simple transformation from Cartesian ones.

If one reads from it the static 4-bein system, one can move to the stationary system

$$\overset{1}{e}_1 = \frac{1}{\alpha}, \quad \overset{2}{e}_2 = \rho, \quad \overset{3}{e}_3 = \alpha\sigma, \quad \overset{4}{e}_3 = -i\alpha\omega\sigma^2, \quad \overset{3}{e}_4 = i\alpha\omega\sigma, \quad \overset{4}{e}_4 = \alpha,$$

$$\text{(L)} \quad \underset{1}{e}^1 = \alpha, \quad \underset{2}{e}^2 = 1/\rho, \quad \underset{3}{e}^3 = \alpha/\sigma, \quad \underset{3}{e}^4 = i\alpha\omega\sigma, \quad \underset{4}{e}^3 = -i\alpha\omega, \quad \underset{4}{e}^4 = 1/\alpha, \tag{5.5}$$

$$\alpha = A/\rho = 1\Big/\sqrt{1-\omega^2\sigma^2}$$

with the help of a generalized Lorentz transformation. (5.5) can be obtained from the Kerr-C system (Burghardt 1982) putting $M = 0$. When calculating the field strengths one has to take into account the differential rotation law:

$$\sigma\omega_{|1} = -2\omega\sigma_1, \quad \sigma\omega_{|2} = 0 \tag{5.6}$$

with

$$\sigma_\alpha = \sigma_{|\alpha} = \frac{1}{\rho}\{r\sin\vartheta, r\cos\vartheta, 0\}, \quad \sigma_\alpha\sigma^\alpha = 1\ . \tag{5.7}$$

The system of the field strengths is analogous to (2.8)-(2.10), but several extra expresions occur using (5.6). For the field equations one obtains a structure similar to (4.3), whereby it is assured that for the rotator with a differential rotation law and at the same time for all rotating gravitational models that include such a law, the relativity of acceleration is guaranteed.

APPENDIX

In the literature on rotating systems often exaggerated views concerning the ability to read from the metric of a physical system, whether it is in a rotating state or not are presented.

Starting from the Euclidean line element in spherical coordinates

$$ds^2 = dr^2 + r^2 d\vartheta^2 + r^2\sin^2\vartheta\, d\varphi^2 + \left(dx^4\right)^2 \tag{A1}$$

we get after a Galilei-like coordinate transformation

$$\varphi = \varphi' + \omega t, \quad t = t' \tag{A2}$$

with the matrix (here all indices are coordinates indices of the Einstein space)

$$\begin{aligned} &A_3^{3'} = 1, \quad A_4^{3'} = i\omega, \quad A_3^{4'} = 0, \quad A_4^{4'} = 1, \\ &A_{3'}^{3} = 1, \quad A_{4'}^{3} = -i\omega, \quad A_{3'}^{4} = 0, \quad A_{4'}^{4} = 1 \end{aligned} \tag{A3}$$

the metric in the form

$$\begin{gathered} ds^2 = dr^2 + r^2 d\vartheta^2 + r^2 \sin^2\vartheta d\varphi'^2 - 2i\omega\sigma^2 d\varphi' dx^4 + \alpha^{-2}\left(dx^4\right)^2, \\ \sigma = r\sin\vartheta, \quad \alpha = \sqrt{1-\omega^2\sigma^2}. \end{gathered} \tag{A4}$$

We do not want to follow the literature concerning the common in assignment of (A1) to the system at rest and (A4) to a rotating one. Both metrics can describe both systems, the transformation (A3) will only lead to a physically meaningless change of the representation. The state of motion is solely determined by the choice of observer systems, thereby only a relative motion is described. As the metric of most rotating gravitational systems is described in the form (A4), we place the form (A4) also for all further considerations, if one demands holonomic coordinates for the reason of calculation processes. We omit the primes which denote the rotating coordinate system and we define the arc lengths $dx^{\hat{m}}$:

$$dx^{\hat{1}} = dr, \quad dx^{\hat{2}} = r\,d\vartheta, \quad dx^{\hat{3}} = r\sin\vartheta\, d\varphi, \quad dx^{\hat{4}} = dx^4, \tag{A5}$$

where we use hats for the indices of the Lorentz space, if a particular distinction is necessary. According to

$$dx^{\hat{m}} = \overset{m}{e}_i dx^i \tag{A6}$$

we read the connection quantities between the arc lengths and the coordinate differentials:

$$\text{(S)} \qquad \overset{1}{e}_1 = 1, \quad \overset{2}{e}_2 = r, \quad \overset{3}{e}_3 = \sigma, \quad \overset{4}{e}_4 = 1 \tag{A7}$$

and we get the non-diagonal metric for the Lorentz space

$$g_{\hat{1}\hat{1}} = g_{\hat{2}\hat{2}} = g_{\hat{3}\hat{3}} = 1, \quad g_{\hat{3}\hat{4}} = -i\omega\sigma, \quad g_{\hat{4}\hat{4}} = \alpha^{-2}, \tag{A8}$$

whereby the latter is related to the Einstein metric by

$$g_{ik} = \overset{m}{e}_i \overset{n}{e}_k g_{\hat{m}\hat{n}} \ . \tag{A9}$$

The missing diagonality of the metric (A8) is accompanied by a lack of orthogonality of the bein vectors lying in the coordinate axes of the system. Both have their origin in the fact that the vectors S are adapted to the rotating coordinate system whose t-lines are not orthogonal to the (r, ϑ, φ)-hypersurfaces. A spacetime splitting on the indices cannot be carried out casually. Therefore we can drop the system S being inappropriate and we split the metric (A4) of the Einstein space in such a way that either one of the two systems (2.6) or (2.7) arise. Now we can easily see that

$$g_{\hat{m}\hat{n}} = \delta_{\hat{m}\hat{n}} \ , \tag{A10}$$

that is, the metric of Lorentz space is diagonal 1 and hence the spacetime splitting can be performed index-based. The systems A and B of Sec. 2 are connected by the generalized Lorentz transformation L according to (3.1). The system B (indices primed) can be converted into the system S with the Galilei transformation G:

$$\overset{m'}{e}_i = A^{m'}_m \overset{m}{e}_i$$

$$\text{(G)} \qquad \begin{aligned} &A^{3'}_{3} = 1, \quad A^{3'}_{4} = i\omega\sigma, \quad A^{4'}_{3} = 0, \quad A^{4'}_{4} = 1, \\ &A^{3}_{3'} = 1, \quad A^{3}_{4'} = -i\omega\sigma, \quad A^{4}_{3'} = 0, \quad A^{4}_{4'} = 1. \end{aligned} \qquad \text{(A11)}$$

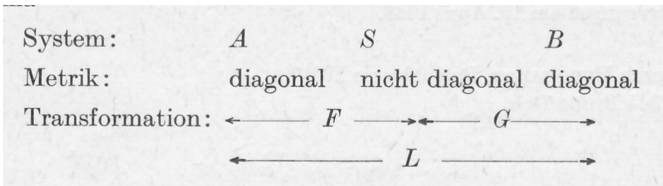
The system A (indices primed) can be converted into the system S with a transformation F which is referred to as "system-conserving" by M. Strauss (1974):

$$\text{(F)} \qquad \begin{aligned} &A^{3''}_{3'} = \alpha, \quad A^{4''}_{3'} = -i\alpha\omega\sigma, \quad A^{3''}_{4'} = 0, \quad A^{4'}_{4} = \alpha^{-1}, \\ &A^{3'}_{3''} = \alpha^{-1}, \quad A^{3'}_{3''} = \ i\alpha\omega\sigma, \quad A^{4'}_{4''} = 0, \quad A^{4'}_{4''} = \alpha. \end{aligned} \qquad \text{(A12)}$$

Both transformations do not lead to any new state of motion, but only to the somewhat unreasonable metric of the Lorentz space (A8), but they are part of the generalized Lorentz transformation

$$L = F \cdot G\,. \qquad \text{(A13)}$$

The correlations may easily be obtained from the scheme



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