ARG Austrian Reports on Gravitation

ARG-2014-03

REMARKS ON THE MODEL OF OPPENHEIMER AND SNYDER IV

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Keywords: gravitational collapse, comoving and non-comoving co-ordinate systems, transformation of systems, decomposition of the field strengths into a circular and a parabolic part

Contents

The model of Oppenheimer and Snyder is geometrically interpreted. It describes a cap of a sphere which slides down on Flamm's paraboloid.

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1. INTRODUCTION

In three previous papers [1-3] we have carried out further investigations on the model of a collapsing stellar object proposed by Oppenheimer and Snyder [4], and we have discovered some surprising properties of the model. In this paper we want to deepen the understanding of the geometric foundations.

2. THE STRUCTURE OF THE OS MODEL

The basis of the interior solution of the OS model is the cap of a sphere which is connected from below to Flamm's paraboloid of the exterior Schwarzschild geometry at a suitable location. During the collapse the cap slides down on Flamm's paraboloid, the matter in the stellar object condenses. During the collapse Flamm's paraboloid remains stable, as prescribed by Birckhoff's theorem.

In [3] we have shown that the field equations and the subequations of the field equations are invariant with respect to a Lorentz transformation that describes a transition from the comoving to the non-comoving system. In this article we want to work out the problem of transformation in more detail to come to a deeper understanding of the processes in the collapse of a star.

The velocity of the particles in the interior of the collapsing star is

$$v = -\frac{r}{\mathcal{R}_g}, \tag{2.1}$$

wherein r is the radial co-ordinate in the embedding space of the cap of a sphere and $\mathcal{R}_g$ the radius of curvature of the cap of a sphere. Th two quantities are related by

$$r = \mathcal{R}_g \sin\eta , \tag{2.2}$$

where the polar angle for the cap of a sphere is η. In the center of the star one has $r = 0$ and $\eta = 0$ and thus the particles are at rest at the center of the collapsing object according to (2.1).

On the surface of the star one has $r = r_g$. This is also the boundary surface of the Schwarzschild geometry surrounding the star. At this location $\mathcal{R}_g$ the radius of curvature of the cap of a sphere and ρ_g the radius of curvature of Flamm's paraboloid lie on a straight line and one has

$$\rho_g = 2\mathcal{R}_g = \sqrt{\frac{2r_g^3}{M}} . \tag{2.3}$$

From this and from (2.1) can be shown that

$$v_g = -\frac{r_g}{\mathcal{R}_g} = -\sqrt{\frac{2M}{r_g}} \quad . \tag{2.4}$$

The velocity v_g of the surface of the star is the collapse velocity of the star. It corresponds to the velocity of an object that has reached the velocity v_g coming from infinity to the position r_g in the Schwarzschild field. Since the collapse velocity is $v_g = 0$ for $r_g \to \infty$ the star was infinitely large at the beginning of the collapse and has filled the newly available space with a Schwarzschild field during the collapse and will reach the velocity of light at $r = 2M$. The model does not allow a shrinking of the star beneath the event horizon. The cap of the sphere would secede from Flamm's paraboloid, the two spaces would not be linked. However, OS have assumed that the star can shrink under the event horizon and then cannot emit light any longer. The OS model has initiated the research on black holes although the term ‘black hole' was invented later on. However, Mitra [5] has shown that a collapse beneath the event horizon is not possible in the OS model. The critical variable in the OS model has the wrong sign.

For the non-comoving system the metric and the 4-bein applies

$$\begin{gathered}
(B) \quad ds^2 = \alpha^2 dr^2 + r^2 d\vartheta^2 + r^2 \sin^2\vartheta \, d\varphi^2 + a_T^2 dit^2 \\
\overset{1}{e}_1 = \alpha, \quad \overset{2}{e}_2 = r, \quad \overset{3}{e}_3 = r\sin\vartheta, \quad \overset{4}{e}_4 = a_T \\
\alpha = \frac{1}{\sqrt{1-\frac{r^2}{\mathcal{R}_g^2}}}, \quad a = \frac{1}{\alpha} = \sqrt{1-\frac{r^2}{\mathcal{R}_g^2}} = \cos\eta, \quad a_T = \alpha\sqrt{\frac{r_g}{2M}\frac{y-1}{y^{3/2}}} \quad . \\
y = \frac{1}{2}\left[\left(\frac{r'}{r'_g}\right)^2 - 1\right] + \frac{r'_g}{2M}\frac{r}{r'}, \quad \alpha_T = 1/a_T
\end{gathered} \tag{2.5}$$

The change of the velocity (2.1) has two reasons. First we envisage a particle at a distinct location and we proceed in the radial direction. We have $r_{|1} = \overset{1}{e}_1 \frac{\partial r}{\partial r} = a$. Since a non-comoving observer does not change its position in time we have $r_{|4} = 0$. Thus, we can write

$$r_{|m} = \{a, 0, 0, 0\}. \tag{2.6}$$

The radius of the cap of the sphere is spatially constant, but due to collapse dependent on the time. Its change is easily computed in the comoving system which we write down in the form of

$$\begin{gathered}
(A) \quad ds^2 = K^2\left[dr'^2 + r'^2 d\vartheta^2 + r'^2 \sin^2\vartheta \, d\varphi^2\right] + dit'^2 \\
\overset{1'}{e}_{1'} = K, \quad \overset{2'}{e}_{2'} = K r', \quad \overset{3'}{e}_{3'} = K r' \sin\vartheta, \quad \overset{4'}{e}_{4'} = 1
\end{gathered} \quad . \tag{2.7}$$

Therein K is the *scale factor*, time-dependent but independent of r', which establishes the connection between the non-comoving and comoving radial co-ordinates

$$r = \mathcal{K} r' \,. \tag{2.8}$$

After some renaming and reshaping the OS approach yields [1,2]

$$\frac{1}{\mathcal{K}}\frac{\partial \mathcal{K}}{\partial r'} = 0, \quad \frac{1}{\mathcal{K}}\frac{\partial \mathcal{K}}{\partial t'} = \mathcal{K}^{\bullet} = -\frac{1}{\mathcal{R}_g}, \quad \frac{\partial \mathcal{R}_g}{\partial r'} = 0, \quad \frac{\partial \mathcal{R}_g}{\partial t'} = \mathcal{R}_g^{\bullet} = -\frac{3}{2}, \quad \frac{\partial r}{\partial r'} = \mathcal{K}, \quad \frac{\partial r}{\partial t'} = v \tag{2.9}$$

with t' as the co-ordinate time of the comoving system. From (2.1) one gets therewith the change of the velocity of the particles in the comoving system inside the stellar object

$$v_{|1'} = -\frac{1}{\mathcal{K}}\frac{\partial}{\partial r'}\frac{r}{\mathcal{R}_g} = -\frac{1}{\mathcal{R}_g}, \quad v_{|4'} = -\frac{\partial}{i\partial t'}\frac{r}{\mathcal{R}_g} = iv\frac{1}{\mathcal{R}_g} - i\frac{r}{\mathcal{R}_g}\frac{1}{\mathcal{R}_g}\mathcal{R}_g^{\bullet} = iv\frac{1}{\mathcal{R}_g} - 3iv\frac{1}{\rho_g} \,. \tag{2.10}$$

In addition, (2.3) was used. The two systems (A) and (B) are connected by the Lorentz transformation

$$L^{1}_{1'} = \alpha, \quad L^{1}_{4'} = -i\alpha v, \quad L^{4}_{1'} = i\alpha v, \quad L^{4}_{4'} = \alpha, \quad \alpha = 1/a \,. \tag{2.11}$$

The velocity v is defined by (2.1) and the Lorentz factor α is identical with the metric factor in (2.5). With this one can convert the rate of change of (2.10) into the non-comoving system. We write the quantities in the form

$$v_{|m'} = a^{C}_{m'} + a^{P}_{m'}, \quad v_{|m} = a^{C}_{m} + a^{P}_{m} \,. \tag{2.12}$$

Thus, we have split the velocity change into a circular and into a parabolic part. The two components

$$\begin{aligned} a^{C}_{m'} &= \{\alpha, 0, 0, -i\alpha v\}\left(-\frac{a}{\mathcal{R}_g}\right), \quad a^{P}_{m'} = \{0,0,0,1\}\left(-3iv\frac{1}{\rho_g}\right) \\ a^{C}_{m} &= \{1,0,0,0\}\left(-\frac{a}{\mathcal{R}_g}\right), \quad a^{P}_{m} = \{-i\alpha v, 0, 0, \alpha\}\left(-3iv\frac{1}{\rho_g}\right) \end{aligned} \tag{2.13}$$

can be understood if one faces a geometric interpretation of the OS model.

The interior OS solution, as mentioned above, can be represented by the geometry of the cap of a sphere. According to (2.1) and (2.2) the inwardly directed velocity

$$v = -\sin\eta \,. \tag{2.14}$$

If one considers the collapse at a time $t' = \text{const.}$ the circular part of the velocity change corresponds to a change of the polar angle, that is, a change in position of an observer in the interior. The parabolic part is due to the decrease of the spherical cap. It includes the radius of curvature ρ_g of the Schwarzschild parabola on the boundary surface, and the typical factor 3 for a parabolic structure. According to these embodiments we are prepared to examine the structure of the quantities of the model and their transformation behavior.

3. THE TRANSFORMATION OF THE FIELD STRENGTHS

The Ricci-rotation coefficients of a gravitational model transform in general inhomogeneously

$$
\begin{aligned}
{}'A_{m'n'}{}^{s'} &= L^{m\;n\;s'}_{m'n's}A_{mn}{}^{s} + {}'L_{m'n'}{}^{s'} \\
A_{mn}{}^{s} &= L^{m'n'\,s}_{m\;n\;s'}\,{}'A_{m'n'}{}^{s'} + L_{mn}{}^{s}
\end{aligned} \quad . \tag{3.1}
$$

The respective last terms in (3.1) are the Lorentz terms which are defined by

$$
{}'L_{m'n'}{}^{s'} = L^{s'}_{s}L^{s}_{n'|m'}, \quad L_{mn}{}^{s} = L^{s}_{s'}L^{s'}_{n|m}, \quad {}'L_{m'n'}{}^{s'} = -L^{m\;n\;s'}_{m'n's}L_{mn}{}^{s} \ . \tag{3.2}
$$

With (2.11) one can write

$$
\begin{aligned}
{}'L_{4'1'}{}^{4'} = {}'L_{1'} = i\alpha^2 v_{|4'}, &\quad {}'L_{1'4'}{}^{1'} = {}'L_{4'} = -i\alpha^2 v_{|1'} \\
L_{41}{}^{4} = L_{1} = -i\alpha^2 v_{|4}, &\quad L_{14}{}^{1} = L_{4} = i\alpha^2 v_{|1}
\end{aligned} \tag{3.3}
$$

and thus, one can convey the Lorentz term clearly as

$$
{}'L_{m'n'}{}^{s'} = h_{m'}{}^{s'}\,{}'L_{n'} - h_{m'n'}\,{}'L^{s'}, \quad L_{mn}{}^{s} = h_{m}{}^{s}L_{n} - h_{mn}L^{s} \ . \tag{3.4}
$$

Therein

$$
h_{mn} = \begin{pmatrix} 1 & & & \\ & 0 & & \\ & & 0 & \\ & & & 1 \end{pmatrix} \tag{3.5}
$$

is a submatrix of the metric.

We also want to decompose the Lorentz terms into circular and into parabolic parts. We first define two circular and parabolic quantities respectively

$$
\begin{aligned}
L^{C}_{m} = \{1,0,0,0\}\left(\alpha v \frac{1}{\mathcal{R}_g}\right), &\quad L^{P}_{m} = \{\alpha,0,0,i\alpha v\}\left(-3\alpha^2 v \frac{1}{\rho_g}\right) \\
{}'L^{C}_{m'} = \{\alpha,0,0,-i\alpha v\}\left(-\alpha v \frac{1}{\mathcal{R}_g}\right), &\quad {}'L^{P}_{m'} = \{1,0,0,0\}\left(3\alpha^2 v \frac{1}{\rho_g}\right)
\end{aligned} . \tag{3.6}
$$

From (2.7) we derive the field strengths

$$'U_{1'} = 'A_{4'1'}{}^{4'} = -\overset{4'}{e}_{4'}\, e^{4'}_{4'|1'} = 0, \quad 'U_{4'} = 'A_{1'4'}{}^{1'} = -\overset{1'}{e}_{1'}\, e^{1'}_{1'|4'} = \frac{1}{\mathcal{K}}\mathcal{K}_{|4'} \tag{3.7}$$

which can be written down for the non-comoving system using (2.11) as well

$$'U_{m'} = \{0,0,0,1\}\frac{i}{\mathcal{R}_g}, \quad 'U_m = \{-i\alpha v, 0, 0, \alpha\}\frac{i}{\mathcal{R}_g}, \quad 'U_m = L_m^{m'}\,'U_{m'}\ . \tag{3.8}$$

Since the stellar object of the OS model collapses in free fall, no gravitation or acceleration forces are acting on the comoving observer in the radial direction but tidal forces are acting. This is recognized from (3.7), first relation. We also note that the quantity 'U of the comoving system has no parabolic part. We now rely on the results (2.13) of the previous Section and we use them in (3.3). Thus, we obtain the desired decomposition of the Lorentz terms

$$L_m = \overset{C}{L}_m + \overset{P}{L}_m - 'U_m\,, \quad 'L_{m'} = '\overset{C}{L}_{m'} + '\overset{P}{L}_{m'} + 'U_{m'}\,, \quad '\overset{C}{L}_{m'} = -L^m_{m'}\overset{C}{L}_m\,, \quad \overset{P}{L}_m = -L_m^{m'}\,'\overset{P}{L}_{m'}\ . \tag{3.9}$$

In a previous paper we have noted that the Ricci-rotation coefficients contain the lateral field quantities B, and C

$$B_{m'} = 'A_{2'm'}{}^{2'} = \left\{\frac{1}{r}, 0, 0, -i\frac{v}{r}\right\}, \quad C_{m'} = 'A_{3'm'}{}^{3'} = \left\{\frac{1}{r}, \frac{1}{r}\cot\vartheta, 0, -i\frac{v}{r}\right\}\ . \tag{3.10}$$

They transform as vectors between the systems (A) and (B). They are circular as well, and they have identical 4th components with 'U

$$'U_{4'} \overset{*}{=} B_{4'} \overset{*}{=} C_{4'} = \frac{i}{\mathcal{R}_g}\ . \tag{3.11}$$

On account of the absence of the parabolic part in the field strengths, the possibility is limited for the comoving observer to inspect the geometric mechanism of the collapse. This is reserved for the non-comoving observer. If we isolate the purely spatial parts of the field quantities by means of

$$^*B_{\alpha'} = \left\{\frac{1}{r}, 0, 0\right\}, \quad ^*C_{\alpha'} = \left\{\frac{1}{r}, \frac{1}{r}\cot\vartheta, 0\right\}, \quad \alpha' = 1,2,3 \tag{3.12}$$

we obtain for the Ricci and the Einstein tensor for these quantities

$$^*R_{\alpha'\beta'} = 0, \quad ^*G_{\alpha'\beta'} = 0\,. \tag{3.13}$$

For the comoving observer, the spatial part of the geometry *appears* flat. But that this is not the case as we have shown in [3]. We process the remaining components of the 4-dimensional quantities to

$$
\begin{gathered}
{}'A_{m'n'}{}^{s'} = {}^*A_{m'n'}{}^{s'} + Q_{m'}{}^{s'}\,{}'u_{n'} - Q_{m'n'}\,{}'u^{s'}, \quad {}'A_{s'n'}{}^{s'} = {}'A_{n'} = {}^*A_{n'} + Q_{s'}{}^{s'}\,{}'u_{n'} \\
Q_{[m'n']} = 0, \quad Q_{m'n'}\,{}'u^{n'} = 0
\end{gathered} \; . \tag{3.14}
$$

The quantities (3.12) are included in $^*A$, and the new symmetric quantity is defined by

$$
Q_{1'1'} = {}'U_{4'}, \quad Q_{2'2'} = B_{4'}, \quad Q_{3'3'} = C_{4'} \; . \tag{3.15}
$$

The unit vector ${}'u^{m'} = \{0,0,0,1\}$ is orthogonal to the surfaces $t' = \text{const.}$ of the interior geometry, it is the rigging vector of this shrinking surface.

On the basis of

$$
{}'u_{m'||n'} = Q_{m'n'} \tag{3.16}
$$

the Q are the second fundamental forms of the surface theory. For the Ricci then only remains

$$
\begin{aligned}
R_{m'n'} = &-\left[Q_{m'n'\wedge s'}\,{}'u^{s'} + Q_{m'n'}Q_{s'}{}^{s'}\right] \\
&-{}'u_{n'}\left[Q_{s'}{}^{s'}{}_{\wedge \underline{m}'} - Q_{m'}{}^{s'}{}_{\wedge s'}\right] \\
&-{}'u_{m'}\left[{}^*A_{n'|s'}\,{}'u^{s'} + Q_{r'}{}^{s'}\,{}^*A_{s'n'}{}^{r'}\right] \\
&-{}'u_{m'}\,{}'u_{n'}\left[Q_{s'}{}^{s'}{}_{\wedge r'}\,{}'u^{r'} + Q_{r's'}Q^{r's'}\right]
\end{aligned} \; . \tag{3.17}
$$

Therein is ${}'u_{m'} = \{0,0,0,1\}$ and

$$
\Phi_{\underline{m}'\wedge \underline{n}'} = \Phi_{\underline{m}'|\underline{n}'} - {}^*A_{n'm'}{}^{s'}\Phi_{\underline{s}'}, \quad \underline{m}' = 1', 2', 3' \tag{3.18}
$$

the 3-dimensional covariant derivative of a 3-dimensional quantity.

The third brackets in the above block vanish identically and the contracted Codazzi equation

$$
Q_{s'}{}^{s'}{}_{\wedge \underline{m}'} - Q_{m'}{}^{s'}{}_{\wedge s'} = 0 \tag{3.19}
$$

in the second row of the block as well. Both relations express that in the comoving system must be $R_{\underline{m}'4'} = 0$.

From this one can calculate the Einstein tensor

$$
G_{\alpha'\beta'} = 0, \quad \kappa\mu_0 = -Q_{\alpha'\beta'}Q^{\alpha'\beta'}, \quad \alpha' = 1', 2', 3' \; . \tag{3.20}
$$

The mass density is made up of the field energy density. From (3.15) and (3.11) one gets for it the more familiar term

$$\kappa\mu_0 = \frac{3}{\mathcal{R}_g^2} = \frac{6M}{r_g^3} \tag{3.21}$$

with r_g the value of r associated to the surface of the collapsing star. Since the star collapses in free fall from infinity, the mass density at the beginning of the collapse is zero due to $r_g \to \infty$. On the other hand is

$$Q_{1'1'} = -i\frac{1}{\mathcal{K}}\mathcal{K}^{\bullet}, \quad Q_{2'2'} = Q_{3'3'} = -i\frac{1}{r}r^{\bullet} \ . \tag{3.22}$$

If one turns off the master switch and thus stops the collapse, then the energy density disappears and hence the mass disappears. The star vanishes and the space is empty. From the perspective of a fictional observer the space appears flat. One can say that the star of the OS model is gaining its mass only from the collapse.

4. THE NON-COMOVING SYSTEM

In the last section we have brought the transformation quantities for the field strengths in a form which reveal their origin. They have a circular part that refers to the cap of a sphere and a parabolic component. The latter arises because the cap of a sphere is sliding down Flamm's paraboloid. The transformation we now want to use is to bring the field quantities of the non-comoving system into relation with those of the comoving system.

First, we have to derive the field quantities from the metric (B) Eq. (2.5). With $y_{|1} = 0$ we obtain with (2.13)

$$U_1 = A_{41}{}^4 = -\overset{4}{e}{}_4 \underset{4|1}{e}{}^4 = \frac{1}{a_T} a_{T|1} = U_1^C + U_1^P, \quad U_4 = A_{14}{}^1 = -\overset{1}{e}{}_1 \underset{1|4}{e}{}^1 = \frac{1}{\alpha}\alpha_{|4} = U_4^C + U_4^P .$$

Combining this we have

$$\begin{aligned} U_m^C &= \{1,0,0,0\}\,\alpha v \frac{1}{\mathcal{R}_g}, \qquad U_m^P = \{\alpha,0,0,i\alpha v\}\left(-3\alpha^2 v \frac{1}{\rho_g}\right) \\ U_{m'}^C &= \{\alpha,0,0,-i\alpha v\}\,\alpha v \frac{1}{\mathcal{R}_g}, \quad U_{m'}^P = \{1,0,0,0\}\left(-3\alpha^2 v \frac{1}{\rho_g}\right) \end{aligned} \tag{4.1}$$

and we have already performed a separation into a circular and into a parabolic part.

The lateral field quantities are related to the spherical cap by

$$B_m = A_{2m}{}^2 = \left\{\frac{a}{r},0,0,0\right\}, \quad C_m = A_{3m}{}^3 = \left\{\frac{a}{r},\frac{1}{r}\cot\vartheta,0,0\right\} . \tag{4.2}$$

They transform into the system (A) as vectors

$$B_{m'} = L^{m}_{m'}B_m, \quad C_{m'} = L^{m}_{m'}C_m \tag{4.3}$$

and take values as described in (3.11) and (3.12). But the U-quantities transform inhomogeneously. With (3.4) the relation (3.1) can be written down for the U-quantities as follows

$$h_{m'}{}^{s'}\,'U_{m'} - h_{m'n'}\,'U^{s'} = L^{m\,n\,s'}_{m'n's}\left[h_m{}^s U_m - h_{mn}U^s\right] + h_{m'}{}^{s'}\,'L_{n'} - h_{m'n'}\,'L^{s'} . \tag{4.4}$$

A similar structure applies to the unprimed quantities. Thus, one gets the simple relations

$$'U_{m'} = U_{m'} + 'L_{m'}, \quad U_m = 'U_m + L_m , \tag{4.5}$$

where we have already calculated the Lorentz quantities L in (3.6) and (3.9) and where we have also separated them into a circular and into a parabolic part. It turns out that the two components transform separately. Taking into account (3.9) one can rearrange (4.5)

$$\begin{aligned} 'U_{m'} &= \left[U^C_{m'} + 'L^C_{m'}\right] + \left[U^P_{m'} + 'L^P_{m'}\right] + 'U_{m'} \\ 'U_m &= \left[U^C_m - L^C_m\right] + \left[U^P_m - L^P_m\right] + 'U_m \end{aligned} . \tag{4.6}$$

It can be seen with (3.6) and (4.1) that the brackets vanish and thus (4.5) is satisfied. From the relations of the non-co-moving system one can recognize the motion of the interior geometry on Flamm's paraboloid. In the comoving system only the tidal forces (3.11) remain after an inhomogeneous transformation.

We have discussed the field equations for both systems in detail in [3]. Here we are only interested in how the stress-energy-momentum tensor arises from the field quantities.

We prepare the problem by decomposing the field quantities in such a way as to make it as easy as possible to calculate the stress-energy-momentum tensor. First, we need $\frac{1}{\alpha}\alpha_{|m} = \alpha^2 v v_{|m}$. With (2.13) and (2.3) we have

$$\frac{1}{\alpha}\alpha_{|1} = -\alpha v\frac{1}{\mathcal{R}_g} - 3\alpha^3 v^3\frac{1}{\rho_g}, \quad \frac{1}{\alpha}\alpha_{|4} = -3i\alpha^3 v^2\frac{1}{\rho_g} .$$

Thus, we again have decomposed a quantity into a circular and into a parabolic part

$$\frac{1}{\alpha}\alpha_{|m} = E^C_m + E^P_m$$
$$E^C_m = \left\{-\alpha v \frac{1}{\mathcal{R}_g}, 0,0,0\right\}, \quad E^P_m = \left\{-3\alpha^3 v^3 \frac{1}{\rho_g}, 0, 0 - 3i\alpha^3 v^2 \frac{1}{\rho_g},\right\} \quad . \tag{4.7}$$

In addition, one takes the quantity U^P_1 from (4.1). With (3.21) one finally has

$$\begin{aligned}
(B_1 + C_1)E^P_1 &= 2\frac{a}{r}\left(-3\alpha^3 v^3 \frac{1}{\rho_g}\right) = \alpha^2 v^2 \kappa\mu_0 = -\kappa T_{11} \\
(B_1 + C_1)E^P_4 &= 2\frac{a}{r}\left(-3\alpha^3 v^2 \frac{1}{\rho_g}\right) = i\alpha^2 v^2 \kappa\mu_0 = -\kappa T_{14} \quad . \\
-(B_1 + C_1)U^P_1 &= 2\frac{a}{r}\left(3\alpha^3 v \frac{1}{\rho_g}\right) = -\alpha^2 \kappa\mu_0 = -\kappa T_{44}
\end{aligned} \tag{4.8}$$

The left-hand terms of the above expression are included in the Einstein tensor. It can be seen that the parabolic parts of the field quantities have a significant role in establishing the stress-energy-momentum tensor. The remaining terms in the Einstein tensor are canceled.

We consider a snapshot of the spatial 3-dimensional part of the model during the collapse at an arbitrary time in the non-comoving system. One has the metric

$${}'ds^2 = \alpha^2 dr^2 + r^2 d\vartheta^2 + r^2 \sin^2\vartheta d\varphi^2 \quad . \tag{4.9}$$

By that we separate the time-dependent quantities that describe the collapse from those describing the body geometry. For the Ricci then only remains

$$\begin{aligned}
{}^*R_{\alpha\beta} = &-\left[B_{\beta\underset{2}{||}\alpha} + B_\beta B_\alpha\right] - b_\alpha b_\beta \left[B^\gamma{}_{\underset{2}{||}\gamma} + B^\gamma B_\gamma\right] \\
&-\left[C_{\beta\underset{3}{||}\alpha} + C_\beta C_\alpha\right] - c_\alpha c_\beta \left[C^\gamma{}_{\underset{2}{||}\gamma} + C^\gamma C_\gamma\right]
\end{aligned} \tag{4.10}$$

with $\alpha = 1,2,3$ and

$$B_{\beta\underset{2}{||}\alpha} = B_{\beta|\alpha}, \quad C_{\beta\underset{3}{||}\alpha} = C_{\beta|\alpha} - B_{\alpha\beta}{}^\gamma C_\gamma \quad . \tag{4.11}$$

With the lateral field quantities defined in (4.2) we obtain after a simple calculation

$${}^*R_{\alpha\beta} = \frac{2}{\mathcal{R}^2_g} g_{\alpha\beta} \, . \tag{4.12}$$

This corresponds to a geometry with constant positive curvature, which we have already interpreted as the cap of a sphere.

However, if we take a look at the corresponding Eq. (3.13) in the comoving system, we can see a contradiction. The spatial geometry appears flat in this system. One could have the impression that the geometry of space can be modified by choosing an appropriate observer system. The problem we have discussed in [3] in detail.

We refer to a basic statement of Einstein: The space can be made locally flat by a suitable co-ordinate transformation at any point of the space. In modern notation this means that the space can be made locally flat by a suitable choice of the reference system at any point of the space. A transformation to such a reference system is a Lorentz transformation. The new reference system is the comoving one. It moves in free fall. For example, one obtains for the lateral field quantities

$$B_{1'} = L_{1'}^{1} B_1 = \alpha \frac{a}{r} = \frac{1}{r}$$

the flat expression $1/r$ according to (4.2) and (4.3), although the space will not be modified by the Lorentz transformation. Only a pseudo-rotation takes place in the tangent planes of the space. Since the Lorentz factor α and the metric factor a satisfy the condition $\alpha a = 1$ we obtain the well-known expression for the quantity B of the flat geometry.

5. SUMMARY

In this paper we have shown that the OS model has a well-defined geometrical basis. The metric of the interior geometry is that of a cap of a sphere which adjoins Flamm's paraboloid of the Schwarzschild exterior solution. During the collapse the spherical cap shrinks and slides down Flamm's paraboloid. Both, the geometric properties of the cap of the sphere and the parabolic properties of the exterior geometry enter into the equations of the model. We have separated the two parts in the formulae and thus we can conclude that the parabolic parts are responsible for the properties of the collapse.

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