ARG Austrian Reports on Gravitation

ARG-2004-01

KERR GEOMETRY IV. ACCELERATED SYSTEMS

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Keywords: Kerr metric, freely falling system, accelerated systems, analogy to electromagnetism, Lorentz invariance

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Abstract: To investigate the field equations of the Kerr model and their invariance properties by use of a freely falling system we start with a flat rotating model and simulate a freely falling system by means of accelerated observers. We find similarities to the theory of the electrodynamics of moving bodies.

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1. INTRODUCTION

To present the basic structure of a freely falling system we use a flat rotating model endowed with a spherical reference system. In Sec. 2 rigid rotating frames are introduced by an active Lorentz transformation. Although the space is flat, the dynamics of this system is treated in terms of the Riemannian geometry. This provides a close relation to the rotating gravitational models. The equations for the rotational fields, derived from the vanishing Ricci tensor, are relativistic generalizations of the equations for classical rotating systems and they exhibit a similarity to the equations of the electrodynamics. As the Ricci equations are a set of non-linear equations, the sources of the fields are quadratic in the field strengths.

In Sec. 3 we perform an additional Lorentz transformation with non-constant velocity vectors. Thus, we will be able to simulate a freely falling system, if we demand the observers' velocities to point towards the center of the rotation. We show that the field equations for the rotating system are invariant under this passive Lorentz transformation.

In Sec. 4 we make a more general ansatz for the acceleration. We decompose the field equations with respect to the accelerated system and we find the field equations to be invariant under active Lorentz transformations. We also elaborate some similarities to the theory of moving bodies in the theory of electrodynamics.

2. A SIMPLIFIED MODEL

In the end, our aim is to study the field equations of the Kerr metric for freely falling observers. To get a plain formalism for this problem we simplify the model. We start with a flat geometry and we use a spherical reference system with respect to the first three dimensions. The 4-bein reads as

$$\overset{1}{e}_1 = 1, \quad \overset{2}{e}_2 = r, \quad \overset{3}{e}_3 = \sigma, \quad \overset{4}{e}_4 = 1, \quad \sigma = r\sin\vartheta . \tag{2.1}$$

By using four orthogonal unit vectors

$$m_n = \{1,0,0,0\}, \quad b_n = \{0,1,0,0\}, \quad c_n = \{0,0,1,0\}, \quad u_n = \{0,0,0,1\} \tag{2.2}$$

we obtain the connexion coefficients

$$A_{mn}{}^{s} = B_{mn}{}^{s} + C_{mn}{}^{s}, \quad B_{mn}{}^{s} = b_m B_n b^s - b_m b_n B^s \quad C_{mn}{}^{s} = c_m C_n c^s - c_m c_n C^s$$
$$B_n = \left\{\frac{1}{r}, 0,0,0\right\}, \quad C_n = \left\{\frac{1}{r}, \frac{1}{r}\cot\vartheta, 0,0\right\} \quad . \tag{2.3}$$

The field equations for these quantities are derived from $R_{mn} \equiv 0$ and describe the curvature of the slices of the spheres

$$B_{\underset{2}{n||m}} + B_n B_m = 0, \quad C_{\underset{3}{n||m}} + C_n C_m = 0 \,. \tag{2.4}$$

Here the graded covariant derivatives [1] have been used. We perform an *active* Lorentz transformation with constant angular velocity ω

$$L_3^{3'} = \alpha_R, \quad L_4^{3'} = i\alpha_R \omega\sigma, \quad L_3^{4'} = -i\alpha_R\omega\sigma, \quad L_4^{4'} = \alpha_R, \quad \alpha_R = 1\Big/\sqrt{1-\omega^2\sigma^2} \,. \tag{2.5}$$

In the new system, the rotational effects can be observed as Coriolis and centrifugal forces. The resulting anholonomic bein-vectors are defined by

$$\begin{aligned} &\overset{3'}{e}_3 = \alpha_R \sigma, \quad \overset{3'}{e}_4 = i\alpha_R\omega\sigma, \quad \overset{4'}{e}_3 = -i\alpha_R\omega\sigma^2, \quad \overset{4'}{e}_4 = \alpha_R \\ &\underset{3'}{e}^3 = \frac{\alpha_R}{\sigma}, \quad \underset{3'}{e}^4 = i\alpha_R\omega\sigma, \quad \underset{4'}{e}^3 = -\, i\alpha_R\omega \,, \quad \underset{4'}{e}^4 = \alpha_R \end{aligned} \tag{2.6}$$

as an additional structure in the flat geometry. The co-ordinate system is still the static one. The covariant derivative transforms under (2.5) as

$$\Phi_{m'||n'} = L^{m\,n}_{m'n'}\Phi_{m||n} = \Phi_{m'|n'} - A_{n'm'}{}^{s'}, \quad A_{n'm'}{}^{s'} = L^{n\,m\,s'}_{n'm's} A_{nm}{}^{s} + L^{s'}_{s} L^{s}_{m'|n'} \,. \tag{2.7}$$

If we evaluate the new connexion with (2.5) and if we drop the primes, we obtain

$$\begin{aligned} &A_{mn}{}^{s} = B_{mn}{}^{s} + C_{mn}{}^{s} + F_{mn}{}^{s}, \quad B_{mn}{}^{s} = b_m B_n b^s - b_m b_n B^s, \quad C_{mn}{}^{s} = c_m {}^*C_n c^s - c_m c_n {}^*C^s \\ &{}^*C_n = \alpha_R^2 C_n = C_n + F_n, \quad F_{mn}{}^{s} = F_{mn}u^s + F^s{}_m u_n + F^s{}_n u_m, \quad F_{mn} = H_{mn} + F_{[m}u_{n]} \quad . \\ &H_{mn} = 2\, i\alpha_R^2 \omega\sigma_{[m}c_{n]}, \quad F_n = \alpha_R^2\omega^2\sigma\sigma_n, \quad \sigma_n = \sigma_{|n} \end{aligned} \tag{2.8}$$

The $u_m = \{0,0,0,1\}$ are the 4-velocities of the rotating observers. The new quantities H_{mn}, F_m are relativistic generalizations of the Coriolis and centrifugal forces of the rigid rotation. Using the techniques as known from general relativity, they appear dynamically in the field equations. For these new field strengths the field equations decouple from the Ricci tensor. Using the fourth[1] graded derivative for the new quantities

$$\Phi_{m\underset{4}{||}n} = \Phi_{m|n} - {}'A_{nm}{}^{s}\Phi_{s}, \quad {}'A_{nm}{}^{s} = B_{nm}{}^{s} + C_{nm}{}^{s} \tag{2.9}$$

we obtain with ${}'R_{mn} = {}'R_{mn}({}'A)$

$$'R_{mn} + 2F_{[m\cdot n\cdot}{}^{s}{}_{\underset{4}{||}s]} - \left[F_{rm}{}^{s}F_{sn}{}^{r} - F_{mn}{}^{s}F_{rs}{}^{r}\right] = 0\,. \tag{2.10}$$

As the rotation is independent of the time, the first term on the right side of

$$2F_{[m\cdot n\cdot}{}^{s}{}_{\underset{4}{||}s]} = F_{mn\underset{4}{||}s}u^{s} + 2u_{(m}F^{s}{}_{n)\underset{4}{||}s} + F_{n\underset{4}{||}m} \tag{2.11}$$

vanishes and we get from (2.10) the set of equations

$$\begin{array}{ll} B_{n\underset{2}{||}m} + B_{n}B_{m} = 0, & B^{s}{}_{\underset{2}{||}s} + B^{s}B_{s} = 0 \\ {}^{*}C_{n\underset{3}{||}m} + {}^{*}C_{n}{}^{*}C_{m} + 3H_{n3}H_{m3} = 0, & {}^{*}C^{s}{}_{\underset{3}{||}s} + {}^{*}C^{s}{}^{*}C_{s} + 3H^{s3}H_{s3} = 0 \\ F_{n\underset{4}{||}m} - F_{n}F_{m} + H_{n}{}^{s}H_{ms} = 0, & F^{s}{}_{\underset{4}{||}s} - F^{s}F_{s} + H^{rs}H_{rs} = 0 \\ \multicolumn{2}{c}{H^{s}{}_{n\underset{4}{||}s} - 2H^{s}{}_{n}F_{s} = 0} \end{array}\,. \tag{2.12}$$

The last two equations can be written as

$$F^{s}{}_{n\underset{4}{||}s} = J_{n}, \quad J_{n} = 2H^{s}{}_{n}F_{s} + \frac{1}{2}\left(F^{s}F_{s} - H^{sr}H_{sr}\right)u_{n}\,, \tag{2.13}$$

or by use of the total covariant derivative as

$$F^{s}{}_{n||s} = 0\,. \tag{2.14}$$

The current is conserved

[1] For time-independent quantities the fourth graded derivative corresponds to the three dimensional space-like covariant derivative.

$$J^m{}_{||m} = 0 . \tag{2.15}$$

From the symmetry properties of the Riemann tensor

$$R_{[mns]}{}^r = 0, \quad R_{[mns]}{}^r u_r = 2u_{[s||mn]} = 0 \tag{2.16}$$

we obtain a second set of equations

$$F_{[mn||s]} = 0 \;\Rightarrow\; H_{[mn\underset{4}{||}s]} = 0, \quad F_{[m\underset{4}{||}n]} = 0 . \tag{2.17}$$

In symbolic notation we write with $i\mathbf{H} \triangleq \frac{1}{2}\varepsilon^{mns}H_{ns}$ the above formulae as

$$\begin{aligned} &\operatorname{div}\mathbf{F} = \mathbf{F}^2 + \mathbf{H}^2, \quad \operatorname{rot}\mathbf{H} = 2(\mathbf{F}\times\mathbf{H}) \\ &\operatorname{div}\mathbf{H} = 0, \qquad\qquad \operatorname{rot}\mathbf{F} = 0 \end{aligned} . \tag{2.18}$$

$$\operatorname{div} 2\left(\mathbf{F}\times\mathbf{H}\right) = 0, \quad \frac{1}{2}\left(\mathbf{F}^2 + \mathbf{H}^2\right)^{\cdot} = 0 . \tag{2.19}$$

For the relativistic case these equations have been investigated by the author in an earlier paper [3,4] and for the non-relativistic case by F. Hund [2]. As the similarity to the electrodynamics is evident, one also speaks of gravito-magnetism [9,10]. In the last decades, many authors have treated this problem.

Comparing the first equation in (2.18) with Newton’s law $\operatorname{div} g = -4\pi k\mu$ we find the source of the centrifugal force to be the density of the rotational field energy $\mu = -\frac{1}{4\pi k}\left(\mathbf{F}^2 + \mathbf{H}^2\right)$. It is the mass density generated by the relatively rotating universe. This mass density is negative and repulsive and its action is very much stronger than the action of the distant rotating mass shell as described by H. Thirring [5] and J. Lense and H. Thirring [6]. As the velocity of light is a constant of the transformation (2.5), we do not suppose that the constancy principle of light is violated in accelerated frames. In [5] we have shown that the phase shift in the interferometer on Sagnac’s platform can be deduced in full agreement with the principle of constancy. The rotational field strengths shorten and lengthen the optical paths of the light beams on the rotating platform. Thus, Sagnac’s experiment is the pendant to the Michelson experiment also concerning its interpretation. Recently the problem of the constant velocity of light in rotating systems has been discussed by E. Minguzzi [7] and E. Minguzzi and A. Macdonald [8].

3. INTRODUCING AN ACCELERATED SYSTEM

For the purpose of studying a freely falling reference system we introduce an observer field with a non-constant velocity v_S. To provide a better comparison to the electrodynamics of moving bodies we perform most of our calculations in a more general way but we finally restrict ourselves to an irrotational motion of the system. To begin with, we assume that v_S depends on the radial positions of the observers and increases towards the center of rotation. Firstly, we describe the physics of the stationary observers in terms of the accelerated system. This will be done by a *passive* Lorentz transformation

$$L_1^{1'} = \alpha_S, \quad L_4^{1'} = i\alpha_S v_S, \quad L_1^{4'} = -i\alpha_S v_S, \quad L_4^{4'} = \alpha_S, \quad \alpha_S = 1\Big/\sqrt{1-v_S{}^2}\,. \tag{3.1}$$

The components of the radial tangent vector and the four-velocity of the stationary systems in terms of the accelerated observers are

$$m_{s'} = \left\{\alpha_S, 0, 0, -i\alpha_S v_S\right\}, \quad u_{s'} = \left\{i\alpha_S v_S, 0, 0, \alpha_S\right\} \tag{3.2}$$

and the partial derivatives

$$\partial_{1'} = \alpha_S \partial_1, \quad \partial_{4'} = -i\alpha_S v_S \partial_1. \tag{3.3}$$

The components of the field strengths read as

$$\begin{aligned} B_{m'} &= \left\{\alpha_S\ B_1,\ 0,\ 0, -i\alpha_S v_S\ B_1\right\} \\ {}^*C_{m'} &= \left\{\alpha_S {}^*C_1, {}^*C_2, 0, -i\alpha_S v_S {}^*C_1\right\} \\ F_{m'} &= \left\{\alpha_S\ F_1,\ F_2,\ 0, -i\alpha_S v_S\ F_1\right\} \\ H_{1'3'} &= \alpha_S H_{13},\ H_{2'3'} = H_{23},\ H_{4'3'} = -i\alpha_S v_S H_{13} \end{aligned} \quad . \tag{3.4}$$

There is no doubt that the field equations are invariant under the Lorentz transformation:

$$R_{m'n'} = L_{m'n'}^{m\ n} R_{mn} = 0\,. \tag{3.5}$$

If we apply (3.1) to (3.5) and if we use the new definitions

$$\Phi_{m'\underset{1}{||}n'} = \Phi_{m'|n'} - L_{n'm'}{}^{s'}\Phi_{s'}, \quad L_{n'm'}{}^{s'} = L_s^{s'} L_{m'|n'}^{s}, \quad A_{n'm'}{}^{s'} = L_{n'm's}^{n\ m\ s'} A_{nm}{}^{s} \tag{3.6}$$

we find

$$R_{m'n'} = A_{m'n'}{}^{s'}{}_{\underset{1}{\|} s'} - A_{n' \underset{1}{\|} m'} - A_{r'm'}{}^{s'} A_{s'n'}{}^{r'} + A_{m'n'}{}^{s'} A_{s'} \,. \qquad (3.7)$$

The above covariant derivative is the first graded derivative [1] in the 'freely falling' system. It leaves the autoparallelism of the unit vectors invariant

$$\begin{gathered} m_{m' \underset{1}{\|} n'} = m_{m'|n'} - L_{n'm'}{}^{s'} m_{s'} = 0 \\ u_{m' \underset{4}{\|} n'} = u_{m'|n'} - \left(L_{n'm'}{}^{s'} + B_{n'm'}{}^{s'} + C_{n'm'}{}^{s'} \right) u_{s'} = u_{m'|n'} - L_{n'm'}{}^{s'} u_{s'} = 0 \end{gathered} \,. \qquad (3.8)$$

The new quantity L can be written as

$$L_{n'm'}{}^{s'} = m_{n'} M_{m'} m^{s'} - m_{n'} m_{m'} M^{s'}, \quad M_{m'} = \left\{ 0,0,0, \frac{1}{v_S} v_{S|4'} \right\}, \qquad (3.9)$$

where $M_{4'}$ describes the change of the velocity v_S measured with the proper time of the 'freely falling' system. The total covariant derivative

$$\Phi_{m'\|n'} = \Phi_{m' \underset{1}{\|} n'} - A_{n'm'}{}^{s'} \Phi_{s'} \qquad (3.10)$$

is composed of two tensorial parts: the first part is the first graded derivative (3.6) and reduces to the ordinary partial derivative only for special reference systems in a similar way, as the ordinary covariant derivative in flat space reduces to the partial one, if a Cartesian system is chosen. The second part consisting of the connexion coefficients, behaves under Lorentz transformations also as a tensor. The connexion coefficients refer to an invariant geometrical structure. In our simplified model, these are the curvatures of the slices of the sphere[2], namely $|B| = 1/r$, $|C| = 1/r \sin\vartheta$ and the additional structure implemented by (3.5). In a gravitational model, as described by the Kerr metric, there is one more curvature vector because the physical surface is curved into higher dimensions. All these properties of the space are not affected by a passive rotation of the reference system in the tangent space. The field equations $R_{m'n'} = 0$ of the simplified model are invariant under passive Lorentz transformations. However, the question is, whether the subequations (2.12) are invariant too. Inserting (3.4) into (3.7) we find with some algebra the same equations (2.12) for the primed system. This demonstrates that an accelerated observer can predict the physics of the stationary observer. For the Kerr metric this calculations are more tedious, but they will show that a gravitational model is invariant under passive Lorentz transformations. Moreover, the model is also invariant with respect to the subequations of the field equations.

[2] In the flat model, we put in by hand this structure; in the gravitational model, the curvatures are a consequence of embedding the physical surface in a higher dimensional space.

4. APPLYING AN ACTIVE LORENTZ TRANSFORMATION

We omit the restrictions we have made in the last chapter and assume the velocity of the accelerated observers to have an arbitrary direction and to be time-dependent. This enables us to compare the theory of an accelerated system in rotating frames with the theory of moving bodies in the electrodynamics. The Lorentz transformation reads as[3]

$$L_{\alpha}^{\beta'} = \delta_{\alpha}^{\beta} + (\alpha+1) e^{\beta} e_{\alpha}, \quad L_{\alpha}^{4'} = -i\alpha_S v_{\alpha}^{S}, \quad L_{4}^{\beta'} = i\alpha_S v_{S}^{\beta}, \quad L_{4}^{4'} = \alpha_S, \tag{4.1}$$

where the e’s are the unit vectors in the direction of the observers’ velocities. If we interpret (4.1) as an active Lorentz transformation, we have to decompose the field equations by using the transformed bein vectors. The components of the 4-velocity of the accelerated observers in the stationary reference system and in the accelerated reference system respectively are

$$'u_{n} = \left\{ -i\alpha_S v_{\alpha}^{S}, \alpha_S \right\}, \quad 'u_{n'} = \{0,0,0,1\} \tag{4.2}$$

We obtain the field strengths $h_{m'n'}$, $f_{m'}$ measured by the accelerated observers in their own system from the relation

$$\begin{aligned} f_{m'n'} &= h_{m'n'} + f_{[m'}\,'u_{n']}, \qquad f_{m'n'}\,'u^{n'} = \frac{1}{2} f_{m'} \\ F_{m'n'} &= H_{m'n'} + F_{[m'}\, u_{n']}, \qquad F_{m'n'} u^{n'} = \frac{1}{2} F_{m'} \, . \\ f_{m'n'} &= F_{m'n'} \end{aligned} \tag{4.3}$$

h and f are three-dimensional quantities with respect to the accelerated system but have time-like components with respect to the stationary system. With the help of (4.2) we get

$$\begin{aligned} h_{\sigma'} &= H_{\sigma'} - \frac{1}{2} \alpha_S \, \varepsilon_{\sigma'}{}^{\alpha'\beta'} v_{\alpha'} F_{\beta'} \\ f_{\sigma'} &= \alpha_S F_{\sigma'} - \alpha_S^2 F_{\beta'} v^{\beta'} v_{\sigma'} + 2\alpha_S \, \varepsilon_{\sigma'}{}^{\alpha'\beta'} v_{\alpha'} H_{\beta'} \end{aligned} \, . \tag{4.4}$$

With the relation

$$\Phi_{\alpha'} = \left[\delta_{\alpha}^{\beta} + (\alpha_S - 1) e^{\beta} e_{\alpha} \right] \Phi_{\beta}, \tag{4.5}$$

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[3] Greek indices are running from 1 to 3.

we are able to express the new quantities by the old ones and we are able to decompose them into parallel and normal components

$$\mathbf{h}^{\parallel} = \mathbf{H}^{\parallel}, \quad \mathbf{h}^{\perp} = \alpha_S \left[\mathbf{H}^{\perp} - \frac{1}{2} \mathbf{v} \times \mathbf{F} \right]. \qquad \mathbf{f}^{\parallel} = \mathbf{F}^{\parallel}, \quad \mathbf{f}^{\perp} = \alpha_S \left[\mathbf{F}^{\perp} + 2 \mathbf{v} \times \mathbf{H} \right] \tag{4.6}$$

These equations exhibit the structure of Minkowski's electrodynamics of moving bodies. To get the field equations for the values measured by the accelerated observers with respect to their own system, we insert the first relation of (4.3) into

$$f^{s'}{}_{m'\|s'} = 0, \quad f^{s'}{}_{m'\underset{4}{\|}s'} = j_{m'} \ . \tag{4.7}$$

The current is split into

$$j_{n'} = {}^*j_{n'} + {}'u_{n'} j_{4'}, \quad {}^*j_{\alpha'} = L^{\beta}_{\alpha'} J_{\beta} + i\alpha_S v^S_{\alpha} J_4, \quad J_4 = J_s u^s, \quad j_{4'} = j_s {}'u^s = \alpha_S \left[J_4 - i J_{\beta} v^{\beta}_S \right] \tag{4.8}$$

or symbolically

$$\mathbf{j}^{\parallel} = \alpha_S \left[\mathbf{J}^{\parallel} + i v_S J \right], \quad \mathbf{j}^{\perp} = \mathbf{J}^{\perp}. \tag{4.9}$$

Our field equations have contributions from the accelerations

$${}'u_{m'\underset{4}{\|}s'} {}'u^{s'} = -a_{m'}, \quad a_{m'} = \left\{ a_{\alpha'}, 0 \right\} \tag{4.10}$$

whereas the total acceleration is

$${}'u_{m'\|s'} {}'u^{s'} = f_{m'} - a_{m'} . \tag{4.11}$$

The field equations resulting from (4.7) and

$$f_{[m'n'\|s']} = f_{[m'n'\underset{4}{\|}s']} = 0 \tag{4.12}$$

$$
\begin{gathered}
h^{\beta'}{}_{\underset{4}{\alpha'\|\beta'}} - \frac{1}{2}\left[f_{\underset{4}{\alpha'\|s'}}\,{}'u^{s'} - f^{\beta'}\,{}'u_{\underset{4}{\alpha'\|\beta'}} + f_{\alpha'}\,{}'u^{s'}{}_{\underset{4}{\|s'}} \right] = j_{\alpha'}, \quad \frac{1}{2}\left[f^{s'}{}_{\underset{4}{\|s'}} + f^{s'} a_{s'} \right] = j \\
i\left[h_{\underset{4}{\sigma'\|s'}}\,{}'u^{s'} + h_{\sigma'}\,{}'u^{s'}{}_{\underset{4}{\|s'}} - h^{\beta'}\,{}'u_{\underset{4}{\sigma'\|\beta'}} \right] + \frac{1}{2}\varepsilon_{\sigma'}{}^{\alpha'\beta'}\left[f_{\underset{4}{\beta'\|\alpha'}} + f_{\alpha'} a_{\beta'} \right] = 0, \quad h^{\beta'}{}_{\underset{4}{\|\beta'}} = 0
\end{gathered}
\tag{4.13}
$$

contain the quantities (h,f) in contrast to Minkowski’s electrodynamics wherein the field equations are formulated with (h,F) and (H,f). This mixed representation is called artificial by A. Sommerfeld in his textbook [11].

In a subsequent paper, we will apply these results to the Schwarzschild metric by adapting the accelerated system to the radially freely falling reference system. Therefore, we hope to be able to study the interaction of the acceleration with the gravity and to gain better insight to the related problem of the Kerr metric.

Acknowledgements: I am indebted to Prof. H.-J. Treder for his kind interest in this work.

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