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Kerr GEOMETRY III. A FIRST STEP TO FIVE DIMENSIONS

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Abstract: We embed the space-like part of the Kerr metric in a higher dimensional flat space by the use of an extra dimension. The field equations decompose into equations for the curvatures of different slices of the geometry.

Keywords: Kerr metric, embedding, curvature vectors, higher dimensions

Contents

I. INTRODUCTION

In previous papers [1, 10] we have rearranged the Kerr metric in such a manner that we were able to identify physically relevant quantities. In this paper we will investigate the possibilities to embed the space-like part of the Kerr metric in a higher dimensional space by adding one dimension to the underlying theory.

In Sec. 2 we will show that most of the information we need for this aim can be drawn from the radial part of the Kerr metric. We will find an expression analogous to the velocity of a freely falling object in the Schwarzschild theory, which is in close relation to the slope of the embedded surface.

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In Sec. 3 we calculate the higher dimensional connexion coefficients, which include some additional anholonomic components.

In Sec. 4 we introduce the methodology of double surfaces. This is necessary for the explanation of the Kerr metric as the basis for a geometrical theory.

In Sec. 5 we deduce the field equations from a single surface. Moreover, in Sec. 6 and we will deduce three-dimensional equations from the double surface theory.

II. ONE ADDITIONAL DIMENSION

We have shown in [1] that for a flat Kerr-like model much of the information necessary to understand this model can be drawn from the radial factor of the metric. Applying this strategy to the line element of the Kerr metric in Boyer-Lindquist co-ordinates we extract from

$$dx^{1^2} = \frac{\Lambda^2}{\delta^2} dr^2, \quad \Lambda^2 = r^2 + a^2 \cos^2 \vartheta, \quad \delta^2 = r^2 + a^2 - 2Mr \tag{2.1}$$

a new quantity

$$\alpha_S^2 = \frac{A^2}{\delta^2} = \frac{1}{1 - \frac{2Mr}{A^2}}, \quad a_S = \alpha_S^{-1}. \tag{2.2}$$

Then we obtain for the radial line element

$$dx^1 = \alpha_S a_R dr, \quad a_R = \frac{\Lambda}{A}, \tag{2.3}$$

where a_R is the radial elliptical factor discussed in [1]. α_S can also be written as

$$\alpha_S = \frac{1}{\sqrt{1 - v_S^2}}, \; v_S = -\frac{r}{A}\sqrt{\frac{2M}{r}}. \tag{2.4}$$

r is the minor semi axis of the confocal Boyer-Lindquist ellipses and serves as radial parameter. A is the major semi axis, a is the eccentricity. The quantity v_S differs from the analogous quantity $v = -\sqrt{\frac{2M}{r}}$ of the Schwarzschild theory by the axis ratio of the confocal ellipses. v is the velocity of a freely falling object in the Schwarzschild theory and also can be interpreted as sin ε, the slope of Flamm's paraboloid being tan ε and the orientation of ε is taken to be cw. We introduce one additional dimension normal to the flat elliptical geometry by

$$\frac{dx^{0'}}{dx^{1'}} = -\tan\varepsilon,\ v_S = \sin\varepsilon,\ a_S = \cos\varepsilon\ , \tag{2.5}$$

where $dx^{1'} = a_R dr$ defines the tangent vector field of the hyperbolae. The latter are the orthogonal trajectories of the BL-ellipses [1]. Thus, we get with $\rho_S = \frac{1}{v_{S|1'}}$ the local curvature vector field

$$\rho_S(r,\vartheta) = \Lambda\sqrt{\frac{2r}{M}\frac{r^2+a^2}{r^2-a^2}} \tag{2.6}$$

of a surface which cross-section is squashed since ρ_S depends on ϑ. For a = 0, ρ_S reduces to the Schwarzschild curvature vector $\rho = \sqrt{\frac{2r^3}{M}}$. A surface of revolution for the Kerr metric was found by Sharp [2]. He considered the two-metric for $\vartheta = \pi/2,\ d\vartheta = 0,\ dt = 0$. The geometrical structure of the space-like part of the Kerr metric was illustrated by Enderlein [3].

With $dv_S = \cos\varepsilon\, d\varepsilon = v_{S|1'} dx^{1'}$ we find for the radial tangent vector

$$dx^1 = \rho_S d\varepsilon = \alpha_S a_R dr\ . \tag{2.7}$$

In the following, we restrict ourselves to the space-like part of the Kerr metric. We start with four normal unit vectors in Cartesian co-ordinates $\overset{a'}{e}_{\underline{a}'},\ a',\underline{a}' = 0,1,...,3$ (co-ordinate indices being underlined) and subject them to a local rotation $D(\varepsilon,\theta,\varphi)$:

$$D_a^{a'} = \begin{pmatrix} \cos\varepsilon & -\sin\varepsilon & 0 & 0 \\ \sin\varepsilon\cos\theta & \cos\varepsilon\cos\theta & -\sin\theta & 0 \\ \sin\varepsilon\sin\theta\cos\varphi & \cos\varepsilon\sin\theta\cos\varphi & \cos\theta\cos\varphi & -\sin\varphi \\ \sin\varepsilon\sin\theta\sin\varphi & \cos\varepsilon\sin\theta\sin\varphi & \cos\theta\sin\varphi & \cos\varphi \end{pmatrix} \tag{2.8}$$

and subsequently diagonalize the new 4-bein in adjusting the co-ordinates with $\Lambda(r,\vartheta,\varphi)$

$$\Lambda_{\underline{a}}^{\underline{a}'} = \begin{pmatrix} a_R\cot\varepsilon & -a_R\tan\varepsilon & 0 & 0 \\ \cos\vartheta & \cos\vartheta & -r\sin\vartheta & 0 \\ \frac{r}{A}\sin\vartheta\cos\varphi & \frac{r}{A}\sin\vartheta\cos\varphi & A\cos\vartheta\cos\varphi & -A\sin\vartheta\sin\varphi \\ \frac{r}{A}\sin\vartheta\sin\varphi & \frac{r}{A}\sin\vartheta\sin\varphi & A\cos\vartheta\sin\varphi & A\sin\vartheta\cos\varphi \end{pmatrix}. \tag{2.9}$$

The angle θ used above is the off-axis angle of the curvature vector ρ_E of the BL-ellipses and its relation to the azimuthal angle ϑ is given by

$$\sin\theta = \frac{r}{\Lambda}\sin\vartheta,\quad \cos\theta = \frac{A}{\Lambda}\cos\vartheta\ . \tag{2.10}$$

The space-like part of the Kerr metric,

$$ds^2 = \alpha_S^2 a_R^2 dr^2 + \Lambda^2 d\vartheta^2 + \sigma^2 d\varphi^2, \ \sigma = A \sin\vartheta , \tag{2.11}$$

is calculated with ($\underline{m},\underline{n} = \underline{1},\underline{2},\underline{3}$) as

$$g_{\underline{mn}} = g_{\underline{a'b'}} \Lambda^{\underline{a'b'}}_{\underline{mn}} . \tag{2.12}$$

This co-ordinate system is anholonomic with respect to $\underline{0}'$ since

$$\Lambda^{\underline{0}'}_{[\underline{a}|\underline{b}]} \neq 0 . \tag{2.13}$$

III. DEFINITION OF THE THE CONNEXION COEFFICIENTS

With the help of (2.8), (2.10) and

$$d\theta = -\frac{v_S}{\rho_H} dx^0 - \frac{a_S}{\rho_H} dx^1 + \frac{1}{\rho_E} dx^2 , \tag{3.1}$$

$$\rho_H = -\frac{\Lambda^3}{a^2 \sin\vartheta \cos\vartheta}, \ \rho_E = \frac{\Lambda^3}{Ar} , \tag{3.2}$$

(ρ_H and ρ_E being the curvature vectors of the BL-hyperbolae and BL-ellipses) we calculate the higher dimensional connexion coefficients

$$Y_{abc} = D^{a'}_c D^{a'}_{[b|a]} + D^{a'}_a D^{a'}_{[b|c]} + D^{a'}_b D^{a'}_{[a|c]} \tag{3.3}$$

and obtain

$$\begin{aligned} Y_{ab}{}^c &= M_{ab}{}^c + N_{ab}{}^c + B_{ab}{}^c + C_{ab}{}^c \\ M_{ab}{}^c &= m_a M_b m^c - m_a m_b M^c \\ N_{ab}{}^c &= \tilde{m}_a N_b \tilde{m}^c - \tilde{m}_a \tilde{m}_b N^c \quad , \\ B_{ab}{}^c &= b_a B_b b^c - b_a b_b B^c \\ C_{ab}{}^c &= c_a C_b c^c - c_a c_b C^c \end{aligned} \tag{3.4}$$

where m_a, b_a and c_a are the unit tangent vectors in the local 1-, 2- and 3-direction respectively, while

$$\tilde{m}_a = \left\{ v_S, a_S, 0, 0 \right\} \tag{3.5}$$

is the projection of the 1-tangent vector of the flat horizontal slice onto the local 0- and 1-directions. The quantities

$$M_b = \left\{\frac{1}{\rho_S},0,0,0\right\}, \quad N_b = \left\{0,\frac{1}{\rho_H},0,0\right\}, \quad B_b = \left\{\frac{v_S}{\rho_E},\frac{a_S}{\rho_E},0,0\right\}$$
$$C_b = \frac{1}{\sigma}\sigma_b, \quad \sigma_b = \sigma_{|b} = \{v_S \sin\theta, a_S \sin\theta, \cos\theta, 0\} = \left\{v_S \frac{r}{\Lambda}\sin\vartheta, a_S \frac{r}{\Lambda}\sin\vartheta, \frac{A}{\Lambda}\cos\vartheta, 0\right\} \tag{3.6}$$

are determined by the curvature vectors ρ_S, ρ_H, ρ_E, and σ. More insight to the structure of the geometry is gained from the decomposition

$$D^c_{c'}(\varepsilon,\theta,\varphi) = \bar{D}^c_{c''}(\varepsilon) R^{c''}_{c'}(\theta,\varphi). \tag{3.7}$$

Since

$$D^c_{c'} D^{c'}_{[b|a]} = \bar{D}^c_{c''} \bar{D}^{c''}_{[b|a]} + \bar{D}^{a''b''c}_{a\ b\ c''} R^{c''}_{c'} R^{c'}_{[b''|a'']}, \tag{3.8}$$

we can show that Y is decomposable as

$$Y_{ab}{}^{c} = M_{ab}{}^{c} + \bar{D}^{a''b''c}_{a\ b\ c''} A_{a''b''}{}^{c''}, \tag{3.9}$$

where M is the new quantity describing the local radial curvature. The components of the second term are the projections of the flat BL-geometry onto the local directions. Another possible decomposition of Y exhibits the anholonomic contributions

$$Y_{abc} = g_{ch} \overset{h}{e}_{\underline{h}} \underset{[b|a]}{e^{\underline{h}}} + g_{bh} \overset{h}{e}_{\underline{h}} \underset{[a|c]}{e^{\underline{h}}} + g_{ah} \overset{h}{e}_{\underline{h}} \underset{[b|c]}{e^{\underline{h}}} + \underset{a}{e^{\underline{a}}} \underset{b}{e^{\underline{b}}} \underset{c}{e^{\underline{c}}} \left[\Lambda^{\underline{c}'}_{\underline{c}} \Lambda^{\underline{c}'}_{[\underline{b}|\underline{a}]} + \Lambda^{\underline{c}'}_{\underline{b}} \Lambda^{\underline{c}'}_{[\underline{a}|\underline{c}]} + \Lambda^{\underline{c}'}_{\underline{a}} \Lambda^{\underline{c}'}_{[\underline{b}|\underline{c}]} \right]. \tag{3.10}$$

As a consequence of this, the unit vector in the local 0-direction has the property

$$n_{a|||b} = n_{a|b} - Y_{ba}{}^{c} n_c = -Y_{ba}{}^{0}, \; n_{[a|||b]} \neq 0 \tag{3.11}$$

and thus the embedding is anholonomic [4,5,6]. Setting

$$A_{mn} = n_{m|||n}, \; m = 1,2,3, \tag{3.12}$$

the components of the second fundamental form are obtained as

$$A_{11} = M_0 = \frac{1}{\rho_S}, \quad A_{22} = B_0 = \frac{1}{\rho_E}\sin\varepsilon, \quad A_{33} = C_0 = \frac{1}{\sigma}\sin\varepsilon\sin\theta. \tag{3.13}$$

At this stage of development we recognize that most of the features of the surface theory are exhausted. We have to accept that an interpretation of the Kerr metric as a V_4 embedded in a flat E_5 is impossible. Due to a theorem of Eisenhart [7] any four-dimensional Ricci-flat metric is flat, if it can be embedded in a five-dimensional flat space. To overcome this problem, we have to make use of a new strategy.

IV. INTRODUCING A DOUBLE SURFACE

In former papers [8,9] we showed that the Schwarzschild exterior and interior solution can be embedded in five dimensions by the use of a double surface. If we rotate the Schwarzschild parabola and its evolute (Neil's parabola), we find two correlated surfaces, connected by the curvature vector of the Schwarzschild parabola. Including this surface in the theory and cutting off the extra dimension and everything which is unnecessary for the description of the physics, we obtain the *physical surface*. The curvature tensor of this surface has all the Riemannian properties but it is not the curvature tensor of a V_4. It consists of the remaining components of a tensor describing the curvatures of both the evolvente and the evolute. In a similar way we will treat the Kerr metric. We start with a single surface, a hypersphere with the radius X. The components of the radius vector in rectilinear orthogonal co-ordinates a' with the extra dimension 0' are

$$\begin{aligned} X^{3'} &= X\sin\varepsilon\sin\theta\sin\varphi \\ X^{2'} &= X\sin\varepsilon\sin\theta\cos\varphi \\ X^{1'} &= X\sin\varepsilon\cos\theta \\ X^{0'} &= X\cos\varepsilon \end{aligned} \quad . \tag{4.1}$$

The flat space line element in spherical co-ordinates is

$$ds^2 = dX^2 + X^2 d\varepsilon^2 + X^2\sin^2\varepsilon\, d\theta^2 + X^2\sin^2\varepsilon\sin^2\theta\, d\varphi^2 \tag{4.2}$$

and the connexion coefficients are

$$\begin{aligned} &X_{10}{}^{1} = \frac{1}{X}, \quad X_{20}{}^{2} = \frac{1}{X}, \qquad X_{21}{}^{2} = \frac{1}{X}\cot\varepsilon \\ &X_{30}{}^{3} = \frac{1}{X}, \quad X_{31}{}^{3} = \frac{1}{X}\cot\varepsilon, \quad X_{32}{}^{3} = \frac{1}{X\sin\varepsilon}\cot\theta \end{aligned} \quad . \tag{4.3}$$

Now we deform the spherical surface into a surface described by the previously defined curvature vectors and we expand the center of the sphere to the correlated surface generated by the evolutes. This is done by mean of the projectors

$$\mathfrak{p}_a^b = X^b{}_{|||a} = X^b{}_{|a} + Y_{a0}{}^{b}X, \;\; X^a = \{X,0,0,0\}. \tag{4.4}$$

As the coefficients Y are already known, we derive from (3.4)

$$\begin{aligned} &\mathfrak{p}_0^0 = \frac{X}{\rho_S}, \quad \mathfrak{p}_1^1 = \frac{X}{\rho_S}, \quad \mathfrak{p}_2^2 = \frac{X v_S}{\rho_E}, \quad \mathfrak{p}_3^3 = \frac{X v_S}{\rho_E} a_R^2 \\ &\mathfrak{p}_0^2 = -\frac{X}{\rho_H} v_S^2, \quad \mathfrak{p}_1^2 = -\frac{X}{\rho_H} a_S v_S \end{aligned} \quad . \tag{4.5}$$

The transition to the double surface is performed by the following relations:

$$\partial_a = \mathfrak{p}_a^b \hat{\partial}_b, \quad dX^b = \mathfrak{p}_a^b dx^a, \quad Y_{ab}{}^c = \mathfrak{p}_a^d X_{db}{}^c ,$$
$$\Phi_{,b} = \hat{\partial}_b \Phi = \frac{\partial \Phi}{\partial X^b} \tag{4.6}$$

$$\partial_0 = \frac{X}{\rho_S}\frac{\partial}{\partial X} - v_S \frac{\partial}{\rho_H \partial \theta}, \quad \partial_1 = \frac{\partial}{\rho_S \partial \varepsilon} - a_S \frac{\partial}{\rho_H \partial \theta}, \quad \partial_2 = \frac{\partial}{\rho_E \partial \theta}, \quad \partial_3 = \frac{\partial}{\rho_E \sin\theta \partial \varphi}. \tag{4.7}$$

As $X|_{surface} = \rho_S$, the first two operators reduce to

$$\partial_0 = \frac{\partial}{\partial \rho_S}, \ \partial_1 = \frac{\partial}{\rho_S \partial \varepsilon}, \tag{4.8}$$

for functions defined on the surface. From (3.1) and $\sigma = \alpha_R^2 \rho_E \sin\theta = A \sin\vartheta$ we obtain for functions defined on a flat horizontal slice

$$\partial_0 = v_S \frac{\partial}{a_R \partial r}, \quad \partial_1 = a_S \frac{\partial}{a_R \partial r}, \quad \partial_2 = \frac{\partial}{\Lambda \partial \vartheta}, \quad \partial_3 = \frac{\partial}{\sigma \partial \varphi}. \tag{4.9}$$

The second expression in (4.6) relates (2.11) and (4.2). The last equation (4.6) relates (3.4) and (4.3).

V. THE FIELD EQUATIONS

By the consequent use of operations (4.6) we accomplish an embedding of the space-like part of the Kerr metric in a flat space. The flat curvature tensor in higher dimensions has the form

$$R_{abc}{}^d(X) = 2\left[X_{[b \cdot c \cdot}{}^d{}_{,a]} + X_{[b \cdot c \cdot}{}^f X_{a]f}{}^d + X_{[ba]}{}^f X_{fc}{}^d \right]. \tag{5.1}$$

By projection, we derive the curvature tensor R(Y) of the double surface as

$$\mathfrak{p}_a^g \mathfrak{p}_b^h R_{ghc}{}^d(X) = R_{abc}{}^d(Y)$$
$$R_{abc}{}^d(Y) = 2\left[Y_{[b \cdot c \cdot}{}^d{}_{|a]} + Y_{[b \cdot c \cdot}{}^f Y_{a]f}{}^d + Y_{[ba]}{}^f Y_{fc}{}^d + X_{fc}{}^d \mathfrak{p}^f_{[a|||b]} \right] = 0 . \tag{5.2}$$

We have to pay special attention to the last term, which also can be written as $Y_{gc}{}^d \left(\mathfrak{p}^{-1}\right)_f^g \mathfrak{p}^f_{[a|||b]}$. For a decomposition into local vertical and local horizontal parts we separate the anholonomic contributions in Y as follows:

$$
\begin{aligned}
&Y_{ab}{}^{c} = A_{ab}{}^{c} + {}^{*}N_{ab}{}^{c}\\
&A_{ab}{}^{c} = M_{ab}{}^{c} + {}^{**}N_{ab}{}^{c} + B_{ab}{}^{c} + C_{ab}{}^{c}\\
&{}^{**}N_{ab}{}^{c} = m_a N_b m^c - m_a m_b N^c, \quad N_b = \left\{0,0,\frac{1}{\rho_H},0\right\}\\
&{}^{*}N_{12}{}^{1} = -v_S^2 N_2, \quad {}^{*}N_{12}{}^{0} = a_s v_S N_2, \quad {}^{*}N_{02}{}^{1} = a_s v_S N_2, \quad {}^{*}N_{02}{}^{0} = v_S^2 N_2
\end{aligned}
\quad . \tag{5.3}
$$

The components $A_{mn}{}^{s}$, m=1,2,3, coincide with the connexion coefficients of the space-like parts of the Kerr geometry. By the contraction of (5.2) and the use of (5.3) we get the Ricci tensor as

$$
\begin{aligned}
R_{ab}(Y) = &-\left[M_{b\underset{1}{|||}a} - M_{b\underset{1}{|||}c} m^c m_a + M_b M_a + M_b N_a\right] - m_b m_a \left[M^c{}_{\underset{1}{|||}c} + M^c M_c\right]\\
&-\left[N_{b\underset{2}{|||}a} - N_{b\underset{2}{|||}c} n^c n_a - N_{b\underset{2}{|||}c} m^c m_a + N_b N_a\right] - \tilde{m}_b \tilde{m}_a \left[N^c{}_{\underset{2}{|||}c} + N^c N_c\right]\\
&-\left[B_{b\underset{2}{|||}a} - B_{b\underset{2}{|||}c} b^c b_a + B_b B_a\right] - b_b b_a \left[B^c{}_{\underset{2}{|||}c} + B^c B_c\right]\\
&-\left[C_{b\underset{3}{|||}a} + C_b C_a\right] - c_b c_a \left[C^c{}_{\underset{3}{|||}c} + C^c C_c\right]
\end{aligned}
\quad . \tag{5.4}
$$

Here we made use of the graded derivatives [1]

$$
M_{b\underset{1}{|||}a} = M_{b|a}, \; N_{b\underset{2}{|||}a} = N_{b|a} - M_{ab}{}^{c} N_c, \; C_{b\underset{3}{|||}a} = C_{b|a} - M_{ab}{}^{c} C_c - N_{ab}{}^{c} C_c - B_{ab}{}^{c} C_c . \tag{5.5}
$$

and of the following relations

$$
\begin{aligned}
&2\left(p^{-1}\right)_0^0 p^0_{[0|||1]} = -\frac{1}{\rho_S}\rho_{S|1}, \quad 2\left(p^{-1}\right)_0^0 p^0_{[0|||2]} = -a_S^2 N_2\\
&2\left(p^{-1}\right)_0^0 p^0_{[2|||1]} = -a_S v_S N_2, \quad 2\left(p^{-1}\right)_1^1 p^1_{[2|||0]} = -a_S v_S N_2, \quad 2\left(p^{-1}\right)_1^1 p^1_{[1|||2]} = -v_S^2 N_2
\end{aligned}
\quad . \tag{5.6}
$$

The first two terms in the brackets of (5.4) describe the radial curvature of the surface. They have two components which could be understood by (4.9) and the fact that ρ_S also depends on ϑ:

$$
\frac{\partial}{\partial\rho_S}\frac{1}{\rho_S} + \frac{1}{\rho_S}\frac{1}{\rho_S} = 0, \quad \frac{\partial}{\Lambda\partial\vartheta}\frac{1}{\rho_S} + \frac{1}{\rho_S}\frac{1}{\rho_H} = 0 . \tag{5.7}
$$

As the radial off-axis angle ε is constant with respect to the local 0-direction, but varies in the local 1-direction, we find

$$
v_{S|0} = 0, \quad a_{S|0} = 0, \quad v_{S|1} = \frac{a_S}{\rho_S}, \quad a_{S|1} = -\frac{v_S}{\rho_S} . \tag{5.8}
$$

It can be shown easily that the field equations decouple as follows:

$$\begin{aligned}
&N_{\underset{2}{b|||a}} - N_{\underset{2}{b|||c}} n^c n_a - N_{\underset{2}{b|||c}} m^c m_a + N_b N_a = -b_b b_a \tilde{\Omega}^{3s}\tilde{\Omega}_{s3}, \quad N^c{}_{\underset{2}{|||c}} + N^c N_c = -\tilde{\Omega}^{3s}\tilde{\Omega}_{s3} \\
&B_{\underset{2}{b|||a}} - B_{\underset{2}{b|||c}} b^c b_a + B_b B_a = \tilde{m}_b \tilde{m}_a \tilde{\Omega}^{3s}\tilde{\Omega}_{s3}, \qquad B^c{}_{\underset{2}{|||c}} + B^c B_c = \tilde{\Omega}^{3s}\tilde{\Omega}_{s3} \quad . \\
&C_{\underset{3}{b|||a}} + C_b C_a = 0, \qquad C^c{}_{\underset{3}{|||c}} + C^c C_c = 0
\end{aligned} \tag{5.9}$$

The meaning of $\tilde{\Omega}^{3s}\tilde{\Omega}_{s3}$ was discussed in [1,10]. The equations above are nothing but the projections of the corresponding flat equations, treated in [1] in agreement with (3.9). By a suitable decomposition of (5.4) we arrive at the three-dimensional components of the field equations in the curved space.

VI. THE DIMENSIONAL REDUCTION

Relation (3.9) enables us to reduce the higher dimensional field equations (5.4) or (5.9), respectively to their three-dimensional counterparts. Defining

$$N_{\underset{2}{n||m}} = N_{n|m}, \quad B_{\underset{2}{n||m}} = B_{n|m}, \quad C_{\underset{3}{n||m}} = C_{n|m} - {}^{**}N_{mn}{}^{s}C_s - B_{mn}{}^{s}C_s \tag{6.1}$$

with m,n,s =1,2,3, we arrive at

$$\begin{aligned}
R_{mn}(A) = {} & m_m m_n (B_0 C_0 + M_0 C_0) + b_m b_n (M_0 B_0 + B_0 C_0) + c_m c_n (M_0 C_0 + B_0 C_0) \\
& + (m_m m_n + b_m b_n) v_S^2 \tilde{\Omega}^{3s}\tilde{\Omega}_{s3} - (m_m m_n + c_m c_n) v_S^2 N^s C_s
\end{aligned}, \tag{6.2}$$

where the left hand term is defined as

$$R_{mn}(A) = A_{mn}{}^{s}{}_{|s} - A_{n|m} - A_{rm}{}^{s} A_{sm}{}^{r} + A_{mn}{}^{s} A_s \,. \tag{6.3}$$

This shows that the related three-dimensional space is not Ricci-flat. Including the remaining dimensions and implementing the rotation will remedy this problem.

VII. CONCLUSIONS

All the field equations we derived from the space-like part of the Kerr geometry are expressions for the curvature vectors of different slices of the geometry. The equation for the 'radial' curvature ρ_S emerges only in the higher dimensional representation of the field equations. We will demonstrate in a subsequent paper that this quantity is important for the time-like part of the Kerr metric and for the formulation of the force of gravity.

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