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KERR GEOMETRY II. PREFERRED REFERENCE SYSTEMS

Rainer Burghardt*

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We reformulate the Kerr model with the help of covariant tetrad formalism, so that the theory appears in a Maxwell-like manner. We study three preferred reference systems.

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* e-mail: arg@wavenet.at, home page: http://arg.or.at/

1. INTRODUCTION

Our aim is to show that we can rearrange the Kerr metric in such a way that it is possible to read off preferred reference systems. From the tetrad connexion we derive tensorial field strengths satisfying covariant field equations. Sets of observers are correlated to these tetrads describing different states of motion in the Kerr field. The question of covariance with respect to tetrad and co-ordinate representation was faced by Treder [1] and the method of observer fields by Hönl und Dehnen [2].

2. SOME PREFERRED SYSTEMS OF REFERENCE

In a former paper [3], we discussed the flat Kerr-like metric based on an elliptical co-ordinate system

$$ds^2 = dx^{1^2} + dx^{2^2} + \left[\alpha_R dx^3 + i\alpha_R \omega\sigma dx^4\right]^2 + \left[-i\alpha_R \omega\sigma dx^3 + \alpha_R dx^4\right]^2, \tag{2.1}$$

$$dx^1 = \frac{\Lambda}{A}dr,\ dx^2 = \Lambda d\vartheta,\ dx^3 = \sigma d\varphi,\ dx^4 = idt,\ \alpha_R = \frac{A}{\Lambda},\ a_R = \frac{\Lambda}{A},\ \omega = \frac{a}{A^2},\ \sigma = A\sin\vartheta, \tag{2.2}$$

$$A^2 = r^2 + a^2,\ \ \Lambda^2 = r^2 + a^2\cos^2\vartheta. \tag{2.3}$$

A and r being the semi-axes of confocal ellipses with eccentricity a, ω the observer's angular velocity, σ the observer's distance from the rotation axis and α_R the Lorentz factor of this rotation. This model has a dynamical implementation of the rotation by a generalized Lorentz transformation. The rotational effects which could be separated from the field equations, are not a geometrical property of the space, but due to a local tetrad transformation. The Kerr metric differs from (2.1) by the occurrence of the 'gravitational' factor, whose geometrical meaning, will be explained in the next paper. With the definitions

$$a_S = \frac{\delta}{A},\ \ \alpha_S = \frac{A}{\delta},\ \ \delta^2 = r^2 + a^2 - 2Mr,\ \ dx^1 = \alpha_S a_R dr,\ \ \partial_1 = a_S \alpha_R \frac{\partial}{\partial r} \tag{2.4}$$

the line element of the Kerr metric reads

$$ds^2 = dx^{1^2} + dx^{2^2} + \left[\alpha_R dx^3 + i\alpha_R \omega\sigma dx^4\right]^2 + a_S^2\left[-i\alpha_R \omega\sigma dx^3 + \alpha_R dx^4\right]^2. \tag{2.5}$$

It is evident that the new metric (2.5) describes a geometry different from (2.1). It is not possible to convert the rotating metric into a static one by a Lorentz transformation. We emphasize that we have defined the circular velocity ωσ upon purely geometrical considerations. The angular velocities do not depend on ϑ, the rotation is rigid on ellipsoidal surfaces r = const.. From (2.5) we read the components of the 4-bein fields. They were used by Carter [4], and we call them the System C:

$$\overset{1}{e}_1 = \alpha_S a_R,\ \overset{2}{e}_2 = \Lambda,\ \overset{3}{e}_3 = \alpha_R\sigma,\ \overset{3}{e}_4 = i\alpha_R\omega\sigma,\ \overset{4}{e}_3 = -ia_S\alpha_R\omega\sigma^2,\ \overset{4}{e}_4 = a_S\alpha_R$$
$$\underset{1}{e}^1 = a_S\alpha_R,\ \underset{2}{e}^2 = \frac{1}{\Lambda},\ \underset{3}{e}^3 = \frac{\alpha_R}{\sigma},\ \underset{3}{e}^4 = i\alpha_R\omega\sigma,\ \underset{4}{e}^3 = -i\alpha_S\alpha_R\omega,\quad \underset{4}{e}^4 = \alpha_S\alpha_R \, . \tag{2.6}$$

From (2.6) we calculate the tetrad connexion:

$$A_{mn}{}^s = B_{mn}{}^s + N_{mn}{}^s + C_{mn}{}^s + H_{mn}{}^s + D_{mn}{}^s + E_{mn}{}^s \, . \tag{2.7}$$

The first three parts of the connexion are space-like and describe the curvature of the ellipses (B), hyperbolae (N) and circles (C) of the ellipsoids and hyperboloids of revolution

$$B_{mn}{}^s = b_m B_n b^s - b_m b_n B^s,\quad N_{mn}{}^s = m_m N_n m^s - m_m m_n N^s,\quad C_{mn}{}^s = c_m C_n^C c^s - c_m c_n C_C^s$$
$$C_n^C = C_n + F_n,\quad F_n = \alpha_R^2\omega^2\sigma\sigma_n \quad , \tag{2.8}$$
$$B_1 = a_S\frac{1}{\Lambda}\Lambda_{|1} = \frac{a_S}{\rho_E},\quad N_2 = \frac{1}{\Lambda}\Lambda_{|2} = \frac{1}{\rho_H},\quad C_1 = \frac{1}{\sigma}\sigma_1 = a_S\frac{r}{A\Lambda},\quad C_2 = \frac{1}{\sigma}\sigma_2 = \frac{1}{\Lambda}\cot\vartheta$$
$$\sigma_1 = \sigma_{|1} = a_S\frac{r}{\Lambda}\sin\vartheta,\ \sigma_2 = \sigma_{|2} = \frac{A}{\Lambda}\cos\vartheta$$

ρ_E and ρ_H being the curvature vectors of the ellipses and hyperbolae [3]. $\{m^s, b^s, c^s, u^s\}$ is the set of unit vectors, u^s also the 4-velocity of the observers. F is the centrifugal field strength and we note the relation $\alpha_R^2 C_n = C_n + F_n$. The mixed quantities are

$$H_{mns} = H_{mn}^C u_s + H_{sm}^C u_n + H_{sn}^C u_m,\quad D_{mns} = D_{mn}^C u_s - D_{sm}^C u_n + \alpha_S D_{[ns]} u_m$$
$$H_{mn}^C = a_s\left(H_{mn} + D_{[mn]}\right),\quad D_{mn}^C = \alpha_S D_{(mn)},\quad H_{mn} = 2i\alpha_R^2\omega\sigma_{[m}c_{n]},\quad D_{mn} = i\alpha_R^2\sigma\,\omega_{|m}c_n \, , \tag{2.9}$$
$$\omega_{|1}\sigma = -2\omega\sigma_{|1},\ \omega_{|2} = 0$$

H_{mn} being the analogue to the classical Coriolis field. Its dual vector $H^m = -i\frac{1}{2}\varepsilon^{mnr}H_{nr}$ has the direction of the axis of rotation. H_{mn}^C is the total rotational field strength and it has a contribution from the differential rotation law. In consequence of the last line of (2.9) H_{13}^C vanishes and therefore the dual vector of H_{mn}^C is normal to the ellipsoids and describes the spin of the observers. As the differential rotating observer field is subject to shears, the field strength D_{mn}^C expresses this action on the observers as can be seen from

$$u_{m||n} = -A_{nm}{}^4 = \Omega_{mn}^C + E_m^C u_n,\quad \Omega_{mn}^C = \Omega_{[mn]}^C + \Omega_{(mn)}^C = -H_{nm}^C - D_{nm}^C \, . \tag{2.10}$$

The last term in (2.7) reads as

$$E_{mn}{}^s = -\left[u_m E_n^C u^s - u_m u_n E_C^s\right],\quad E_n^C = F_n + E_n \, , \tag{2.11}$$

E_n being the gravitational field strength, having only the radial component

$$E_1 = \frac{1}{\alpha_S}\alpha_{S|1} = -\alpha_S\alpha_R\frac{M}{A^4}\left(r^2 - a^2\right) \tag{2.12}$$

and is reduced for a = 0 to the gravitational field strength $E_1 = -\frac{1}{\sqrt{1-2M/r}}\frac{M}{r^2}$ of the Schwarzschild model. From the Lorentz factor of the rotation we derive another quantity D_m, having its origin in the differential rotation law:

$$\frac{1}{\alpha_R}\alpha_{R|m} = F_m + D_m,\ D_m = \alpha_R^2\omega\omega_{|m}\sigma^2. \tag{2.13}$$

A further decomposition of the Kerr metric was utilized by Iyer and Kumar [5]. With the auxiliary quantity $\gamma = \sqrt{\frac{2Mr}{\Lambda^2}}$ the tetrads read as

$$\begin{aligned} &\overset{3}{e}_3 = \gamma\delta\sin\vartheta,\quad \overset{4}{e}_3 = -i\left(\frac{1}{\gamma} - \gamma\right)a\sin\vartheta,\quad \overset{3}{e}_4 = 0,\quad \overset{4}{e}_4 = \frac{1}{\gamma} \\ &\underset{3}{e}^3 = \frac{1}{\gamma\delta\sin\vartheta},\quad \underset{3}{e}^4 = i\left(\frac{1}{\gamma} - \gamma\right)\frac{a}{\delta}\sin\vartheta,\quad \underset{4}{e}^3 = 0,\quad \underset{4}{e}^4 = \gamma \end{aligned}. \tag{2.14}$$

We do not recommend using (2.14) for calculating the field strengths. The new system, that we call system A is related to the system C by a generalized Lorentz transformation

$$\overset{s}{e}_i(A) = A^{s}_{s'}\overset{s'}{e}_i(C),\quad A^3_{3'} = \alpha_{AC},\quad A^3_{4'} = -i\alpha_{AC}\omega_{AC}\sigma,\quad A^4_{3'} = i\alpha_{AC}\omega_{AC}\sigma,\quad A^4_{4'} = \alpha_{AC}, \tag{2.15}$$

$$\omega_{AC} = \alpha_S\omega,\ \alpha_{AC} = 1\big/\sqrt{1-\omega_{AC}^2\sigma^2}. \tag{2.16}$$

$\omega_{AC}\sigma$ is the velocity of the system A relative to the system C. Performing the transformation (2.15) we find $\gamma = \alpha_{AC}\alpha_S a_R$. With the definitions

$$a_{AC} = 1/\alpha_{AC},\ \omega_{BC} = a_S\omega \tag{2.17}$$

we get

$$\begin{aligned} &\overset{3}{e}_3 = \alpha_{AC}a_R\sigma,\quad \overset{4}{e}_3 = i\alpha_{AC}\left(\omega_{AC} - \omega_{BC}\right)\alpha_R\sigma^2,\quad \overset{3}{e}_4 = 0,\quad \overset{4}{e}_4 = a_{AC}a_S\alpha_R \\ &\underset{3}{e}^3 = \frac{1}{\alpha_{AC}a_R\sigma},\quad \underset{3}{e}^4 = -i\alpha_{AC}\left(\omega_{AC} - \omega_{BC}\right)\alpha_S\alpha_R\sigma,\quad \underset{4}{e}^3 = 0,\quad \underset{4}{e}^4 = \alpha_{AC}\alpha_S a_R \end{aligned}. \tag{2.18}$$

The connexion coefficients for the system A are decomposed in a similar way as in (2.7). The first two parts of the connexion remain unchanged. The other parts get a new interpretation:

$$C_{mn}{}^{s} = c_m C_n^A c^s - c_m c_n C_A^s,\quad C_n^A = C_n + \left(F_n^{AC} - F_n\right) + \left(D_n^{AC} - D_n\right)$$
$$F_n^{AC} = \alpha_{AC}^2 \omega_{AC}^2 \sigma\sigma_n,\quad D_n^{AC} = \alpha_{AC}^2 \omega_{AC} \omega_{AC|n} \sigma^2 \qquad (2.19)$$

The mixed components of the connexion with space-like and time-like indices are simpler because the deformations D_{mns} vanish and H_{mns} consists of antisymmetric quantities only:

$$H_{mns} = \Omega_{mn}^A u_s + \Omega_{sm}^A u_n + \Omega_{sn}^A u_m$$
$$\Omega_{nm}^A = \Omega_{mn}^{AC} - \Omega_{mn},\ \Omega_{mn}^{AC} = H_{mn}^{AC} + D_{mn}^{AC},\quad H_{mn}^{AC} = 2i\alpha_{AC}^2 \omega_{AC} \sigma_{[m} c_{n]},\quad D_{mn}^{AC} = 2i\alpha_{AC}^2 \omega_{AC|[m} c_{n]} \sigma \qquad (2.20)$$
$$\Omega_{3n} = -\Omega_{n3} \doteq \Omega_{3n}^C$$

and we find the dual vector of H_{mn}^{AC} to be parallel to the symmetry axis of the ellipsoids of revolution. The motion of the observers is free of shear

$$u_{m||n} = \Omega_{mn}^A + E_m^A u_n,\ \Omega_{(mn)}^A = 0\,. \qquad (2.21)$$

The last part of (2.7) contains the gravitational field strength (2.12) and the centrifugal forces

$$E_{mn}{}^{s} = -\left(u_m E_n^A u^s - u_m u_n E_A^s\right),\quad E_n^A = E_n + \left(F_n^{AC} - F_n\right) + \left(D_n^{AC} - D_n\right). \qquad (2.22)$$

We note the useful relations

$$F_n^{AC} = -i\,\omega_{AC} \sigma H_{n3}^{AC},\quad D_n^{AC} = -i\,\omega_{AC} \sigma D_{n3}^{AC}$$
$$\Omega_{m3}^{AC} = \alpha_{AC}^2 a_R^2 \alpha_S \left(H_{m3} + D_{m3}\right) + i\,\alpha_{AC}^2 \omega_{AC} \sigma E_m \qquad (2.23)$$

The new component of the centrifugal field strength F_n^{AC} is normal to the symmetry axis of the ellipsoids of revolution and repulsive, while D_n^{AC} is normal to the ellipsoids and attractive.

The last system we discuss in this paper is the locally non-rotating system of Bardeen [6], which we call the system B. With the definitions

$$\omega_{BC} = a_S \omega,\quad \alpha_{BC} = 1\Big/\sqrt{1 - \omega_{BC}^2 \sigma^2},\quad a_{BC} = 1/\alpha_{BC} \qquad (2.24)$$

we get

$$\overset{3}{e}_3 = a_{BC} \alpha_R \sigma,\ \overset{4}{e}_3 = 0,\ \overset{3}{e}_4 = i\alpha_{BC} a_S \left(\omega_{AC} - \omega_{BC}\right) \alpha_R \sigma,\ \overset{4}{e}_4 = \alpha_{BC} a_S a_R$$
$$\underset{3}{e}^3 = \frac{1}{a_{BC} \alpha_R \sigma},\ \underset{3}{e}^4 = 0,\ \underset{4}{e}^3 = -i\alpha_{BC} \left(\omega_{AC} - \omega_{BC}\right) \alpha_R,\quad \underset{4}{e}^4 = a_{BC} \alpha_S \alpha_R \qquad (2.25)$$

The velocity ω_{AB} of the system A relative to the system B is calculated by

$$\omega_{AB} = \frac{\omega_{AC} + \omega_{CB}}{1 + \omega_{AC} \omega_{CB} \sigma^2},\ \alpha_{AB} = 1\Big/\sqrt{1 - \omega_{AB}^2 \sigma^2} = \alpha_{AC} \alpha_{CB} \left(1 + \omega_{AC} \omega_{CB} \sigma^2\right),\ \omega_{CB} = -\omega_{BC}\,. \qquad (2.26)$$

Since $1+\omega_{AC}\omega_{CB}\sigma^2 = a_R^2$ our definition of the angular velocity $\omega_{AB} = \dfrac{2Mra}{\delta A \Lambda^2}$ differs from that one of Bardeen. For (2.7) we get new components

$$C_n^B = C_n - \left(F_n^{BC} - F_n\right) - \left(D_n^{BC} - D_n\right), \quad E_n^B = E_n - \left(F_n^{BC} - F_n\right) - \left(D_n^{BC} - D_n\right) \\ F_n^{BC} = \alpha_{BC}^2 \omega_{BC}^2 \sigma\sigma_n, \quad D_n^{BC} = \alpha_{BC}^2 \omega_{BC} \omega_{BC|n} \sigma^2 . \tag{2.27}$$

There is no contribution to H_{mns}, but

$$D_{mns} = -D_{mn}^B u_s + D_{sm}^B u_n + H_{ns}^B u_m, \quad D_{mn}^B = 2d_{(m}c_{n)}, \quad H_{mn}^B = 2d_{[m}c_{n]} \\ d_m = i\alpha_{BC}^2 \omega_{BC} \sigma_m + i\alpha_{BC}^2 \omega_{BC|m} \sigma + \Omega_{3m}^C . \tag{2.28}$$

The symmetric quantity D_{mn}^B describes the shears acting on the observer fields

$$u_{m||n} = D_{mn}^B + E_m^B u_n, \; D_{[mn]}^B = 0 . \tag{2.29}$$

For M = 0 these three systems coincide. This case we have discussed in paper I. All the field strengths defined above satisfy the Einstein vacuum field equations. We will show this in detail in the next chapter.

3. FIELD EQUATIONS AND CONSERVATION LAWS

The Einstein vacuum field equations

$$R_{mn} = A_{mn}{}^{s}{}_{|s} - A_{n|m} - A_{rm}{}^{s} A_{sn}{}^{r} + A_{mn}{}^{s} A_s = 0 \tag{3.1}$$

may be rewritten in such a manner that they may be decomposed into fairly Maxwell-like covariant equations. We have to make use of the graded derivatives [7]

$$\Phi_{m\underset{2}{||}n} = \Phi_{m|n}, \; \Phi_{m\underset{3}{||}n} = \Phi_{m|n} - \left(B_{nm}{}^s + N_{nm}{}^s\right)\Phi_s, \; \Phi_{m\underset{4}{||}n} = \Phi_{m|n} - \left(B_{nm}{}^s + N_{nm}{}^s + C_{nm}{}^s\right)\Phi_s . \tag{3.2}$$

For the system C we obtain for the Ricci tensor

$$
\begin{aligned}
R_{mn} = &-\left[N_{n\underset{2}{||}m} - N_{n\underset{2}{||}s} m^s m_m + N_n N_m \right] - m_n m_m \left[N^s{}_{\underset{2}{||}s} + N^s N_s \right] \\
&-\left[B_{n\underset{2}{||}m} - B_{n\underset{2}{||}s} b^s b_m + B_n B_m \right] - b_n b_m \left[B^s{}_{\underset{2}{||}s} + B^s B_s \right] \\
&-\left[C^C_{n\underset{3}{||}m} + C^C_n C^C_m \right] - c_n c_m \left[C^s_{C\underset{3}{||}s} + C^s_C C^C_s - \Omega^{rs}_C \Omega^C_{sr} \right] \\
&+\left[E^C_{n\underset{4}{||}m} - E^C_n E^C_m \right] + u_n u_m \left[E^s_{C\underset{4}{||}s} - E^s_C E^C_s - \Omega^{rs}_C \Omega^C_{sr} \right] \\
&+ 2u_{(n} \left[\Omega^s_{Cm)\underset{4}{||}s} - 2H^s_{Cm)} F_s \right] - 2\Omega^C_{n3}\Omega^C_{m3}
\end{aligned} \quad . \tag{3.3}
$$

With

$$
E^C_{n\underset{4}{||}m} = E^C_{n\underset{3}{||}m} + c_n c_m C^s_C E^C_s, \quad \Omega^{[ms]}_C = H^{ms}_C, \quad E^C_s = F_s + E_s \tag{3.4}
$$

we are able to rearrange the field equations so that

$$
\begin{aligned}
&\left[N_{n\underset{2}{||}m} - N_{n\underset{2}{||}s} m^s m_m + N_n N_m \right] + b_n b_m \left[B^s{}_{\underset{2}{||}s} + B^s B_s \right] + \\
&+\left[B_{n\underset{2}{||}m} - B_{n\underset{2}{||}s} b^s b_m + B_n B_m \right] + m_n m_m \left[N^s{}_{\underset{2}{||}s} + N^s N_s \right] + \\
&+\left[C^C_{n\underset{3}{||}m} + C^C_n C^C_m \right] - \left[E^C_{n\underset{3}{||}m} - E^C_n E^C_m \right] + 2\Omega^C_{n3}\Omega^C_{m3} = 0 \\
&\left[C^s_{C\underset{3}{||}s} + C^s_C C^C_s - C^s_C E^C_s - \Omega^{rs}_C \Omega^C_{sr} \right] = 0 \\
&\left[E^s_{C\underset{4}{||}s} - E^s_C E^C_s - \Omega^{rs}_C \Omega^C_{sr} \right] = 0 \\
&\left[\Omega^{sm}_{C\ \underset{4}{||}s} + 2\Omega^{[ms]}_C E^C_s \right] = 0
\end{aligned} \quad . \tag{3.5}
$$

To verify this, we evaluate some of the brackets

$$
\begin{gathered}
N_{n\underset{2}{||}m} - N_{n\underset{2}{||}s} m^s m_m + N_n N_m = - b_n b_m \tilde{\Omega}^{s3}\tilde{\Omega}_{3s} \\
B_{n\underset{2}{||}m} - B_{n\underset{2}{||}s} b^s b_m + B_n B_m = m_n m_m a^2_S \tilde{\Omega}^{s3}\tilde{\Omega}_{3s} - E_n B_m \\
N^s{}_{\underset{2}{||}s} + N^s N_s = -\tilde{\Omega}^{s3}\tilde{\Omega}_{3s}, \; B^s{}_{\underset{2}{||}s} + B^s B_s = a^2_S \tilde{\Omega}^{s3}\tilde{\Omega}_{3s} - B^s E_s \\
\left[C^C_{n\underset{3}{||}m} + C^C_n C^C_m \right] - \left[E^C_{n\underset{3}{||}m} - E^C_n E^C_m \right] + 2\Omega^C_{n3}\Omega^C_{m3} = \left(m_n m_m + b_n b_m \right) \left[\left(1 - a^2_S \right) \tilde{\Omega}^{s3}\tilde{\Omega}_{3s} + B^s E_s \right]
\end{gathered} \quad , \tag{3.6}
$$

where the expressions $\tilde{\Omega}^{s3}\tilde{\Omega}_{3s}$ are the contributions of the evolutes of the ellipses of the flat geometry discussed in paper I [3]. The next two equations can be contracted into

$$
C^s_{C||s} - \Omega^{rs}_C \Omega^C_{sr} = 0, \quad E^s_{C||s} - \Omega^{rs}_C \Omega^C_{sr} = 0 \tag{3.7}
$$

or expanded to

$$C^{s}{}_{||s} + F^{s}{}_{||s} - \Omega^{rs}_{C}\Omega^{C}_{sr} = 0, \quad E^{s}{}_{||s} + F^{s}{}_{||s} - \Omega^{rs}_{C}\Omega^{C}_{sr} = 0\,. \qquad (3.8)$$

The second set of field equations is

$$F_{[m\underset{4}{||}n]} + D_{[m\underset{4}{||}n]} = 0, \quad E_{[m\underset{4}{||}n]} = 0, \quad \Omega^{C}_{[mn\underset{4}{||}s]} = \Omega^{C}_{[mn}D_{s]} - \Omega^{C}_{[mn}E_{s]} = 0\,. \qquad (3.9)$$

There are also conservation laws for the field energy and the Poynting vector

$$\left[E^{s}_{C}E^{C}_{s} + \Omega^{sr}_{C}\Omega^{C}_{rs}\right]_{|4} = 0, \quad \left[2\Omega^{[ms]}_{C}E^{C}_{s}\right]_{\underset{4}{||}m} = 0\,. \qquad (3.10)$$

From (3.5) we read the relations $C^{s}{}_{||s} = E^{s}{}_{||s}$, which indicate a closer geometrical connection of the different slices of this geometry.

For the system A we get the same structure (3.5) for the field equations and conservation laws, if we substitute for the covariant derivatives and field strengths the expressions (2.19) - (2.23). Particularly

$$\begin{gathered} E^{s}{}_{||s} + \left[F^{s}_{AC} - F^{s}\right]_{||s} + \left[D^{s}_{AC} - D^{s}\right]_{||s} - \Omega^{rs}_{A}\Omega^{A}_{sr} = 0, \quad \Omega^{sm}_{A\,\underset{4}{||}s} - 2\Omega^{sm}_{A}E^{A}_{s} = 0 \\ F^{AC}_{[m\underset{4}{||}n]} + D^{AC}_{[m\underset{4}{||}n]} = 0, \quad \Omega^{AC}_{[mn\underset{3}{||}s]} = 0 \\ \left[E^{s}_{A}E^{A}_{s} + \Omega^{sr}_{A}\Omega^{A}_{rs}\right]_{|4} = 0, \quad \left[2\Omega^{sm}_{A}E^{A}_{s}\right]_{\underset{4}{||}m} = 0 \end{gathered} \quad . \qquad (3.11)$$

In the same way we treat the system B

$$\begin{gathered} E^{s}{}_{||s} - \left[F^{s}_{BC} - F^{s}\right]_{||s} - \left[D^{s}_{BC} - D^{s}\right]_{||s} - D^{rs}_{B}D^{B}_{sr} = 0, \; D^{sm}_{B\,\underset{4}{||}s} = 0 \\ F^{BC}_{[m\underset{4}{||}n]} + D^{BC}_{[m\underset{4}{||}n]} = 0, \quad \left[E^{s}_{B}E^{B}_{s} + D^{sr}_{B}D^{B}_{rs}\right]_{|4} = 0 \end{gathered} \quad . \qquad (3.12)$$

There is no transport of gravitational energy in the locally non-rotating system.

4. OUTLOOK

Although we have made some progress in understanding the Kerr geometry, we believe that further considerations should be made. The equation (3.6) could not be understood intuitively. In the equations (3.8), (3.11) and (3.12) the separation of the centrifugal and the gravitational parts is tedious. Some improvements could be achieved by introducing an extra dimension.

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