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FLAT OR NOT FLAT, THIS IS THE QUESTION

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Abstract: Comparing Melia's R_h=ct model and our Subluminal Model, regarding the flatness of both the models.

1. INTRODUCTION

In quite a lot of papers on cosmology, there is some misunderstanding about the possible flatness of the universe. In this regard, we will face some problems. So first, we note the basic equations and explain the variables. In Section. 2 we use rather plain words to investigate the problem of flatness.

For an expanding universe, the canonical form of the metric can be written as

$$ds^2 = \frac{1}{1-k\frac{r^2}{\mathcal{R}^2}} dr^2 + r^2 d\Omega^2 + g_{44} dx^{4^2}, \quad dx^4 = i(c)dt. \tag{1.1}$$

Here, k is the curvature parameter, with the values of $k = (1,0,-1)$, possibly indicating a positively curved, a flat, or a negatively curved universe. Ω contains spherical or hyperbolic angular functions. A special case is –

$$ds^2 = \mathcal{K}^2 \left(dr^2 + r^2 d\vartheta^2 + r^2 \sin^2 d\varphi^2\right) - dt^2, \tag{1.2}$$

$\mathcal{K}$ being the time dependent scale factor, r and t coordinates, comoving with the expansion. The curvature parameter is $k = 0$, and the universe is assumed to be flat and infinite. In several papers [1-5], we outlined that $k = 0$ indicates a universe expanding in free fall. Thus, the metric (1.2) describes a locally flat but not a globally flat universe.

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2. EINSTEIN'S ELEVATOR

There are different views concerning the flatness of the universe we are living in. Melia [6] deduces from his R_h=ct model that we are living in a flat infinite universe. Armed with the results of our Subluminal Model we claim that the universe is globally curved, and finite. Since the formal results of these two models are the same, we have the suspicion that both models will agree, if Melia’s model is geometrically reinterpreted. Some work related to this has already been done in preceding papers. In this note, we use plain words to explain this problem. Doing so, we follow a tradition of Einstein and other authors of his time.

Jane is in an elevator falling towards the Earth	
Jane is in Einstein’s elevator with closed windows. She experiences no forces. She hovers. Jane is acquainted with General relativity. She concludes that the space is flat.	Jane assumes the space to be **globally flat.**
She opens the windows, and realizes that the elevator is in free fall towards the Earth. She concludes that the Earth is responsible for gravitational forces acting on the elevator. Thus, the space cannot be flat.	Jane realizes that the space is only **locally flat**.
The elevator is arriving on Earth. Jane experiences forces, and is convinced that the space is curved.	Jane is convinced that the space is **globally curved**.
Jane is in a cosmological elevator	
Jane is living in a universe expanding in free fall. She is following the expansion enclosed in Einstein's cosmological elevator, with closed windows. She does not experience any forces. She concludes that the universe is flat.	Since the scale factor is $k=0$ and the lapse function of the metric is $g_{44}=1$, Jane assumes the universe to be **globally flat**.
She opens the windows and realizes that the elevator follows the expansion in free fall. She assumes that the absence of forces is due to the motion in free fall. Jane concludes that the universe only appears to be flat.	Systems in free fall have $k=0$ and $g_{44}=1$, thus a universal cosmic time. Jane herself is bound to a system *commoving* with free fall. Thus, the universe is only **locally flat**.
Jane leaves the elevator, experiences forces, and is convinced that the universe is curved.	Jane leaves the cosmic elevator using a *Lorentz transformation*, transforming *the comoving* system to a *non-comoving* system. Forces emerge. Jane is convinced that the universe is **globally curved**.

Jack is in a cosmological elevator	
Jack is living in a universe expanding in free fall. He is following the expansion enclosed in Einstein's cosmological elevator with closed windows. He does not experience any force. He concludes that the universe is flat.	Since the lapse function of the metric is $g_{44} = 1$, Jack assumes the universe to be **globally flat**.
He tries to open the windows, but they clamp.	He tried a *coordinate transformation* from a *comoving* system to a *non-comoving* system. But he ended up with an equation containing non-comoving coordinates, but still containing the universal time of the comoving system. He failed.
Jack cannot leave the elevator. He is caught in Einstein's cosmological elevator forever.	Jack believes that he is living in a **globally flat** universe.

3. CONCLUSIONS

Melia [7] tried to leave the elevator using a coordinate transformation, to re-structure the metric in such a way that it appears in Schwarzschild form. This new form of the metric should be interpreted as a metric in non-comoving coordinates. But it contains, besides the non-comoving coordinates, also the cosmic time. Thus, this metric cannot be attributed to a non-comoving observer, and no issue can be made about the curvature of the universe. In contrast, applying a Lorentz transformation [1] to the field strengths written in coordinate invariant tensor form, one finds radial forces driving apart the galaxies. Due to General Relativity, forces arise from the curvature of space. Thus, the universe described by (1.2) is locally flat, but globally curved.

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[6] Most of the papers by Melia and his colleagues are listed in [1]

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