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GEOMETRICAL FOUNDATION OF THE EQUATION OF STATE IN COSMOLOGICAL MODELS

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Abstract: This study demonstrates that the equation of state (EOS) of an expanding cosmological model is derivable from the geometrical properties of the model. We do this using our Subluminal Model, which is geometrically based on a pseudo-hypersphere.

1. INTRODUCTION

The standard model was derived from the pressure-free Friedman model through the insertion of pressure into the stress-energy-momentum tensor. However, we are critical of this approach. The model renders Einstein's field equations underdetermining by extending the right-hand side. Variables in Einstein's field equations appear on the left-hand side, which can be manipulated at will and adapted to astrophysical data. We believe that Nature is not based on inexact solutions of Einstein's field equations. Therefore, we have proposed an exact solution [1] that incorporates pressure and mass density. In addition to previous explanations, we aim to show that the EOS is a property of the underlying geometry.

2. THE EQUATION OF STATE IN THE SUBLUMINAL MODEL

The Subluminal Model is based on the de Sitter universe (dS), omitting the condition $\mathcal{R} = \text{const.}$, where $\mathcal{R}$ is the radius of a pseudo-hypersphere. The latter is the basic structure of both the dS model and our Subluminal Model [1].

The pseudo-hypersphere's design is quite simple: Einstein's field equations decompose into subequations that describe the curvatures of the normal and oblique

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slices of the pseudo-hypersphere and are of the type $\frac{d}{dr}\frac{1}{r}+\frac{1}{r^2}=0$. The structure of the stress-energy-momentum tensor, which also factors in the expansion of the universe, is correspondingly simple:

$$T_{mn}=\begin{pmatrix} -p & & & \\ & -p & & \\ & & -p & \\ & & & \mu_0 \end{pmatrix}. \tag{1}$$

Here, p is the pressure and μ_0 the mass density. The quantities have the values

$$p=-\frac{1}{\mathcal{R}^2},\quad \mu_0=\frac{3}{\mathcal{R}^2}. \tag{2}$$

From this, we can immediately read the EOS

$$\mu_0+3p=0, \tag{3}$$

a relation widely accepted by cosmologists. However, we are not satisfied with this quick result and seek to delve deeper.

We note according to (1)

$$G_{\alpha\beta}=\kappa p g_{\alpha\beta},\quad G_{44}=-\kappa\mu_0,\quad \alpha=1,2,3$$

and

$$\kappa\left(\mu_0+3p\right)=-G^4{}_4+G^\alpha{}_\alpha=0.$$

If we use therein the Ricci tensor and the Ricci scalar, we obtain

$$\left(R^\alpha{}_\alpha-\frac{1}{2}\delta^\alpha_\alpha R\right)-\left(R^4{}_4-\frac{1}{2}R\right)=0,\quad R^\alpha{}_\alpha-R^4{}_4-R=0,$$

finally $R-2R^4{}_4-R=0$, and lastly for the condition for the validity of the equation (3),

$$R_{44}=0. \tag{4}$$

In fact, the following relation applies to the Subluminal Model

$$R_{44}=-\left(U^s{}_{|s}+U^sU_s\right)=0 \tag{5}$$

with[1]

$$U_m=\left\{0,0,0,\frac{1}{\mathcal{R}}\mathcal{R}_{|4}\right\}=\left\{0,0,0,-\frac{i}{\mathcal{R}}\right\}. \tag{6}$$

Equation (5) is the Friedman equation and reads in simplified form

$$\frac{1}{\mathcal{R}^2}\mathcal{R}^{\bullet}-\frac{1}{\mathcal{R}^2}=0,$$

which immediately results in

$$\mathcal{R}^{\bullet}=1,\quad \mathcal{R}^{\bullet\bullet}=0. \tag{7}$$

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[1] We use the original Minkowski notation $x^4=i\left(c\right)t$

If the Subluminal Model is based on an expanding pseudo-hypersphere, the requirement $R_{44} = 0$ leads to the EOS $\mu_0 + 3p = 0$ and at the same time to the condition that the expansion of the universe is unaccelerated ($\ddot{\mathcal{R}} = 0$). The latter is supported by the PLACK project, whose data were analyzed in detail by Melia [2].

Integrating (7), we get

$$\mathcal{R} = (c)t$$

and considering $r = \mathcal{R} \sin\eta$ with η as the polar angle of the pseudo-hypersphere, we obtain at the equator of the pseudo-hypersphere $r_h = \mathcal{R}$, while the relation

$$r_h = ct \tag{8}$$

fixes the cosmic horizon.

Using a different notation, we can write (8) as $R_h = ct$. This expression is the name of an expanding cosmological model proposed by Melia [3,4]. The model is assumed to be flat and infinitely large. In Melia's notation, R is the radial coordinate that does not comove with the expansion, while t is the universal cosmic time in the comoving system. Since Melia's model is globally flat, Melia does not arrive at the expressions (2), which contain the radius of the universe, nor does he arrive at the EOS, Eq. (3). Since he considers an EOS such as (3) to be very useful, he attempts in subsequent papers [5,6] to justify this principle in different ways. What both models have in common, however, is that they are linear models, i.e., neither of their expansions accelerate.

If we use $\mathcal{R} = \mathcal{R}_0 a$ in (7), where $\mathcal{R}_0$ is a constant and *a* is the time-dependent scale factor, we obtain $\ddot{a} = 0$, a relation that Melia requires. In the standard model, an expression containing $\ddot{a}$ is called the deceleration parameter. In any case, Melia's model and our Subluminal Model are equivalent as far as the physical conclusions are concerned. This means that the astrophysical data that Melia has meticulously assembled supports both models equally.

In previous papers [7], we have endeavored to uncover the difference between the two models. The crux of this distinction lies in the interpretation of the curvature parameter k. In both models $k = 0$ is in the comoving coordinate system. For Melia, this means that space is flat, i.e. infinite, whereas we posit that space is locally flat but globally curved. Since the universe expands in free fall, gravitational forces are imperceptible in it, according to Einstein's elevator principle [8]. Therefore, space expanding in free fall *appears* flat to an observer comoving with it.

3. CONCLUSIONS

We have shown that in an expanding cosmological model based on the geometric structure of a pseudo-hypersphere, pressure and mass density can be represented geometrically, leading inevitably to the EOS in the form $\mu_0 + 3p = 0$. Furthermore, we have established a general condition for this form of the EOS, noting its unattainability within a flat model.

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