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ANHOLONOMY AND GRAVITATION

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Abstract: Riemannian geometry is extended with anholonomic structures to provide the possibility to explain the geometrical properties of some gravitational models.

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1. INTRODUCTION

The present paper is a mathematical supplement to the tensor calculus, which can be used in the theory of gravitation. The mentioned extension refers to anholonomy effects, which often only appear hidden in gravitational models. Schouten dealt extensively with anholonomic systems in his book *Ricci-Calculus* [1]. However, the procedures were carried out using the coordinate method.

When gravitation is measuring space, one needs rods and clocks to get physically relevant data. Mathematically, these are represented by tetrads. To compare measured values at different points of space, one needs a law for transporting vectors and tensors in curved space. For this purpose, it is advantageous to use the Ricci-rotation coefficients. Furthermore, it is mandatory to apply the original Minkowski notation with $x^4 = i(c)t$. In contrast, most authors use the coordinate method with Christoffel symbols. In one of our papers [2], we compared the tetrad formalism with the coordinate formalism and applied them to simple geometrical and physical problems. It was easy to see that the components of the Ricci-rotation coefficients describe the curvatures of the normal and inclined slices of surfaces, while the Christoffel symbols were a collection of trigonometric functions with less relation to geometrical and, lastly, to physical properties. Thus, we use the tetrad methodology throughout this paper.

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2. THE METHODS

Tetrads are four orthogonal vectors which provide a decomposition of geometrical and physical quantities into four components that represent measured values. For an arbitrary vector Φ one has

$$\Phi^m = \overset{m}{e}_i \Phi^i, \quad \Phi_m = \underset{m}{e}^i \Phi_i . \tag{2.1}$$

Here, m,… represent tetrad indices, and i,… coordinate indices. The basic relations for tetrads and their relation to the metric are:

$$\overset{m}{e}_i \underset{n}{e}^i = \delta^m_n, \quad g_{mn} = g_{ik} \underset{m}{e}^i \underset{n}{e}^k = \delta_{mn} . \tag{2.2}$$

The tetrad representation has an advantage: dragging of indices is simple:

$$\Phi_m = \Phi^m .$$

Furthermore, in the Minkowski notation, the time-like components of tensors do not change the sign by dragging the associated index of these components.

The covariant derivative of a vector is defined by:

$$\Phi_{m||n} = \Phi_{m|n} - A_{nm}{}^s \Phi_s . \tag{2.3}$$

The Ricci-rotation coefficients are constructed with the tetrads as follows:

$$A_{mn}{}^s = \overset{s}{e}_j \underset{[n|m]}{e}^j + g^{sr} g_{nt} \overset{t}{e}_j \underset{[m|r]}{e}^j - g^{sr} g_{mt} \overset{t}{e}_j \underset{[r|n]}{e}^j, \quad A_{m(ns)} = 0 . \tag{2.4}$$

They have Christoffel symmetry concerning the indices. One has

$$A_{[mn]}{}^s = \overset{s}{e}_j \underset{[n|m]}{e}^j ,$$

followed immediately by:

$$\underset{[n||m]}{e}^i = 0 , \tag{2.5}$$

a relation which is useful for some calculations.

Performing an anholonomic coordinate transformation

$$\Phi_i = \Lambda_i^{i'} \Phi_{i'}, \quad \Phi_{i'} = \Lambda_{i'}^i \Phi_i$$

with $\Lambda_{i'}^i \neq x^i_{|i'}$, the new coordinates i are anholonomic and

$$\Lambda_j^{j'} \Lambda^j_{[k'|i']} \neq 0$$

is Schouten's object of anholonomy. Applying this to the tetrad formalism, one obtains:

$$\Lambda_{mn}{}^s = \underset{m}{e}^{i'} \underset{n}{e}^{k'} \overset{s}{e}_{j'} \Lambda_j^{j'} \Lambda^j_{[k'|i']} , \tag{2.6}$$

the tetrad object of anholonomy.

Transforming the Ricci-rotation coefficients (2.4) into anholonomic coordinates, one obtains:

$$A_{mn}{}^s = {}^*A_{mn}{}^s + \Omega_{mn}{}^s, \quad \Omega_{mns} = \Lambda_{mns} + \Lambda_{smn} - \Lambda_{nsm} . \tag{2.7}$$

Here, $^*A$ is the holonomic part of the Ricci-rotation coefficients, and Ω, the anholonomic supplement which has Christoffel symmetry and the properties

$$\Omega_{m(ns)} = 0, \quad \Omega_{[mn]}{}^{s} = \Lambda_{mn}{}^{s}. \tag{2.8}$$

We consider the commutation relations:

$$\Phi_{|[mn]} = {}^*A_{[nm]}{}^{s}\Phi_{|s}, \quad \Phi_{r|[mn]} = {}^*A_{[nm]}{}^{s}\Phi_{r|s}, \quad \Phi_{||[mn]} = \Lambda_{mn}{}^{s}\Phi_{|s}. \tag{2.9}$$

We define the curvature tensor as follows:

$$\Phi_{r||[mn]} = \frac{1}{2}\hat{R}_{mnr}{}^{s}\Phi_{s} + \Lambda_{mn}{}^{s}\Phi_{r|s}, \quad \hat{R}_{mnr}{}^{s} = 2\left[A_{[n\cdot r\cdot}{}^{s}{}_{||m]} - A_{[n\cdot r\cdot}{}^{t}A_{m]t}{}^{s}\right]. \tag{2.10}$$

Inserting for Φ the tetrads, one obtains:

$$R_{mnr}{}^{s} = 2\overset{s}{e}_{i}\, e^{i}_{r||[mn]}. \tag{2.11}$$

Expanding this relation, one finally gets:

$$\begin{aligned}
R_{rmn}{}^{s} &= 2\left[A_{[m\cdot n\cdot}{}^{s}{}_{||r]} - A_{[m\cdot n\cdot}{}^{t}A_{r]t}{}^{s} + \Lambda_{rm}{}^{t}\overset{s}{e}_{i}\, e^{i}_{n|t}\right] \\
R_{rmn}{}^{s} &= 2\left[A_{[m\cdot n\cdot}{}^{s}{}_{|r]} - A_{[m\cdot n\cdot}{}^{t}A_{r]t}{}^{s} + A_{[mr]}{}^{t}A_{tn}{}^{s} + \Lambda_{rm}{}^{t}\overset{s}{e}_{i}\, e^{i}_{n|t}\right]. \\
R_{mn} &= A_{mn}{}^{s}{}_{||s} - A_{n||m} + A_{sn}{}^{t}A_{mt}{}^{s} - A_{mn}{}^{s}A_{s} + \Lambda_{rm}{}^{s}\overset{r}{e}_{i}\, e^{i}_{n|s} \\
R_{mn} &= A_{mn}{}^{s}{}_{|s} - A_{n|m} + A_{sn}{}^{t}A_{tm}{}^{s} + A_{mn}{}^{s}A_{s} + \Lambda_{rm}{}^{s}\overset{r}{e}_{i}\, e^{i}_{n|s}
\end{aligned} \tag{2.12}$$

The last terms in the above equations vanish for the holonomic case, and the Ricci-rotation coefficients degenerate to holonomic expressions. In this case, the curvature tensors are the ordinary Riemann and Ricci tensors in tetrad notation. The last terms in (2.12) also vanish for stationary models. Since these models are independent of time $\left(x^4 = it\right)$, rotational effects take place in the $\left(x^3 = \varphi\right)$ direction, and in addition, the quantities are independent of t and φ, it turns out that these terms are zero. This drastically simplifies calculations.

Furthermore, the well-known Riemannian relation

$$R_{rmn}{}^{s} + R_{nrm}{}^{s} + R_{mnr}{}^{s} = 0 \tag{2.13}$$

is also valid for the anholonomic case. This can be proofed by inserting (2.11), re-arranging the indices, and applying (2.5). Proofing the Bianchi identities with the conditions mentioned above

$$R_{rmn}{}^{s}{}_{||p} + R_{prn}{}^{s}{}_{||m} + R_{mpn}{}^{s}{}_{||r} = 0 \tag{2.14}$$

is more tedious. Again one has to apply (2.11) and (2.5). Contracting twice one obtains:

$$R_{m}{}^{n}{}_{||n} - \frac{1}{2}R_{||m} = 0. \tag{2.15}$$

According to Einstein's field equations, one gets the conservation laws $T_{m}{}^{n}{}_{||n} = 0$.

We have collected all the formulae we need to develop models with anholonomic features. Now, we are prepared to give an example for such models and refer to models known in the literature in which anholonomy is hidden.

3. A SIMPLE EXAMPLE

A very significant solution of Einstein’s field equations is the Kerr model. It describes the exterior field of a rotating stellar object. It has a hidden anholonomy in the portion of the geometrical structure describing the rotational effects of the field. We drastically simplify the model in such a manner that we can show why and how anholonomy is a necessary feature of the theory.

The Kerr geometry is based on an elliptic-hyperbolic system. We reduce it to a spherical system, and we switch off the gravitational force. Then we are left with a simple spherical line element in flat space:

$$ds^2 = dr^2 + r^2 d\vartheta^2 + r^2 \sin^2\vartheta d\varphi^2 + \left(idt\right)^2, \tag{3.1}$$

and the tetrads associated to this metric are:

$$\begin{array}{llll} \overset{1}{e}_1 = 1, & \overset{2}{e}_2 = r, & \overset{3}{e}_3 = \sigma, & \overset{4}{e}_4 = 1 \\ \underset{1}{e}^1 = 1, & \underset{2}{e}^2 = \dfrac{1}{r}, & \underset{3}{e}^3 = \dfrac{1}{\sigma}, & \underset{4}{e}^4 = 1 \end{array}. \tag{3.2}$$

We use the abbreviation

$$\sigma = r\sin\vartheta. \tag{3.3}$$

Further, priming the coordinate indices in (3.2) and performing the transformation

$$\begin{array}{llll} \Lambda^{3'}_3 = \alpha, & \Lambda^{3'}_4 = i\alpha\omega, & \Lambda^{4'}_3 = -i\alpha\omega\sigma^2, & \Lambda^{4'}_4 = \alpha \\ \Lambda^{3}_{3'} = \alpha & \Lambda^{3}_{4'} = -i\alpha\omega, & \Lambda^{4}_{3'} = i\alpha\omega\sigma^2, & \Lambda^{4}_{4'} = \alpha \end{array}, \tag{3.4}$$

one obtains new tetrads:

$$\begin{array}{llllll} \overset{1}{e}_1 = 1, & \overset{2}{e}_2 = r, & \overset{3}{e}_3 = \alpha\sigma, & \overset{3}{e}_4 = i\alpha\omega\sigma, & \overset{4}{e}_3 = -i\alpha\omega\sigma^2, & \overset{4}{e}_4 = \alpha \\ \underset{1}{e}^1 = 1, & \underset{2}{e}^2 = \dfrac{1}{r}, & \underset{3}{e}^3 = \dfrac{\alpha}{\sigma}, & \underset{3}{e}^4 = i\alpha\omega\sigma, & \underset{4}{e}^3 = -i\alpha\omega, & \underset{4}{e}^4 = \alpha, \end{array} \tag{3.5}$$
$$\alpha = 1\Big/\sqrt{1-\omega^2\sigma^2}$$

which have the structure of the oblique-angled Carter 4-beine of the Kerr model. Since σ and α are functions of r and ϑ, it is evident that $\Lambda^i_{i'} \neq x^i_{|i'}$. Thus, the transformation (3.4) is anholonomic. ω is the constant angular velocity, and $\omega\sigma$, the orbital velocity of a rotating observer. Since we restricted ourselves to $\omega = \text{const.}$, the model represents a rigid rotator, and the orbital velocity will exceed the velocity of light at a critical distance of the rotating observer from the rotation axis. Although this simple model cannot describe Nature, it has some features common with physical models. It is the very simplicity that enables us to work out structures that explain rotational effects of more sophisticated models.

Applying (3.5) to (2.4), for the anholonomic contribution to the Ricci-rotation coefficients, one obtains:

$$\Omega_{mn}{}^{s} = H_{mn}{}^{s} + G_{mn}{}^{s}$$
$$H_{mn}{}^{s} = H_{mn} u^{s} + H^{s}{}_{m} u_{n} + H^{s}{}_{n} u_{m}, \quad G_{mn}{}^{s} = c_{m} F_{n} c^{s} - c_{m} c_{n} F^{s} - u_{m} F_{n} u^{s} + u_{m} u_{n} F^{s} \quad . \tag{3.6}$$

Here,

$$c_{m} = \{0,0,1,0\}, \quad u_{m} = \{0,0,0,1\} \tag{3.7}$$

are unit vectors and

$$H_{mn} = i\alpha^{2}\omega\sigma_{[m}c_{n]}, \quad F_{m} = \alpha^{2}\omega^{2}\sigma\sigma_{m}, \quad \sigma_{m} = \sigma_{|m} = \{\sin\vartheta, \cos\vartheta, 0, 0\}, \quad \sigma^{m}\sigma_{m} = 1 \tag{3.8}$$

are the relativistic generalizations of the Coriolis and centrifugal force.

As mentioned in Sec. 2, the last terms in (2.12) vanish, and the curvature tensors exhibit Riemannian structure, but the Ricci-rotation coefficients still contain anholonomic contributions. It can be shown that the anholonomic structure can be separated from the spherical structure given by (3.1).

Solving the field equations, it is advantageous to perform a [3+1] decomposition and define a 3-dimensional covariant differential operator:

$$\Phi_{m\wedge n} = \Phi_{m|n} - {}'A_{nm}{}^{s}\Phi_{s}, \quad {}'A_{nm}{}^{s} = {}^{*}A_{nm}{}^{s} + \left[c_{m}F_{n}c^{n} - c_{m}c_{n}F^{n}\right]$$
$$A_{nm}{}^{s} = {}'A_{nm}{}^{s} - \left[u_{m}F_{n}u^{n} - u_{m}u_{n}F^{n}\right] + H_{nm}{}^{s} \quad . \tag{3.9}$$

Here, $^{*}A$ represents the Ricci-rotation coefficients of the flat geometry (3.1), describing the curvatures of the grater circles with radii r and the parallels with radii $r\sin\vartheta$ concerning the spherical parametrization of space. Evidently, the associated Ricci vanishes: $^{*}R_{mn} = 0$. Thus, we are left with:

$$\begin{aligned} R_{33} &= -\left[F^{s}{}_{\wedge s} + H^{sr}H_{sr} - F^{s}F_{s}\right] = 0 \\ R_{34} &= H^{s}{}_{3\wedge s} - 2F_{s}H^{s}{}_{3} = 0 \\ R_{44} &= \left[F^{s}{}_{\wedge s} + H^{sr}H_{sr} - F^{s}F_{s}\right] = 0 \end{aligned} \quad . \tag{3.10}$$

This shows that anholonomy does not influence the curvatures of space but implements additional properties in space. With basic calculations, one gets two additional relations:

$$F_{[m\wedge n]} = 0, \quad H_{[mn\wedge s]} = 0 \,. \tag{3.11}$$

Similar structures resulted for the Kerr model, but they are somewhat more complicated. Kerr does not start with the simple Ansatz (3.1) but with an elliptic-hyperbolic system. Furthermore, the lapse function is not chosen to be unity. Thus, gravitational forces emerge and enter the field equation. Moreover, the angular velocity is not constant but is a function of the radial coordinate. Thus, a differential rotation law is implemented into the Kerr model to force the rotation to vanish at infinity. As a consequence, shears enter into the theory; this complicates the model. The corresponding equations for the Kerr metric as shown in [3] are:

$$C^{s}{}_{||s} + F^{s}{}_{||s} - \Omega^{rs}_{C}\Omega^{C}_{sr} = 0, \quad E^{s}{}_{||s} + F^{s}{}_{||s} - \Omega^{rs}_{C}\Omega^{C}_{sr} = 0$$
$$F_{\underset{4}{[m||n]}} + D_{\underset{4}{[m||n]}} = 0, \quad E_{\underset{4}{[m||n]}} = 0, \quad \Omega^{C}_{\underset{4}{[mn||s]}} = \Omega^{C}_{[mn}D_{s]} - \Omega^{C}_{[mn}E_{s]} = 0 \,.$$

They are consequences of an anholonomic transformation generating the Kerr model from a static elliptic-hyperbolic seed metric. We wrote a pedagogical paper [4] to better understand this model.

4. POSSIBLE PHYSICAL APPLICATIONS

To get more insights into the anholonomic contributions to the field equations, we introduce the dual vector to H_{rs} by

$$H_s = -\frac{i}{2}\varepsilon_s{}^{mn}H_{mn} = \{H\cos\vartheta, -H\sin\vartheta\}, \quad H = \alpha^2\omega$$

and obtain in traditional vector notation

$$\begin{aligned} &\operatorname{div}\vec{F} = \vec{F}^2 + 2\vec{H}^2, \quad \operatorname{rot}\vec{F} = 0 \\ &\operatorname{div}\vec{H} = 0, \qquad \operatorname{rot}\vec{H} = 2\vec{H}\times\vec{F} \end{aligned}. \tag{4.1}$$

Although the model we are facing is not physically usable, the anholonomic structure can possibly provide some insights into more sophisticated models. The expression $2\vec{H}\times\vec{F}$ corresponds to the Poynting vector of electrodynamics. The Poynting vector and the energy density of the field are conserved:

$$\operatorname{div}\left(2\vec{H}\times\vec{F}\right) = 0, \quad \left(\vec{F}^2 + 2\vec{H}^2\right)^{\cdot} = 0. \tag{4.2}$$

All these equations have the structure of the Maxwell equations of electrodynamics, with the difference that the field quantities are coupled to themselves. This is the consequence of the non-linearity of the field equations (3.10). Several authors tried a similar decomposition of the field equations and called structures like (4.1) gravitomagnetism.

The quadratic terms in (3.10) can be integrated into the trace free quantity:

$$t_{mn} = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & -\left[H^{sr}H_{sr} - F^sF_s\right] & -2F_sH^s{}_3 \\ & & -2F_sH^s{}_3 & \left[H^{sr}H_{sr} - F^sF_s\right] \end{pmatrix}, \tag{4.3}$$

which satisfies the conservation law,

$$t^m{}_{n||m} = 0, \quad n = 3,4. \tag{4.4}$$

The expression (4.3) consists of field stresses, field current, and field energy. Although this conservation law might not be valid for realistic models, it might give us a hint to solve the outstanding problem of defining and conserving field energy.

Hund [5] set up the equations for a slowly rotating system in an innovative paper on gravitation without using Einstein's field equations. He discussed the non-relativistic ansatz,

$$\vec{F} = \omega^2\sigma\vec{\sigma}, \quad \vec{H} = \vec{\omega}, \quad \operatorname{div}\vec{F} = 2\omega^2, \quad \operatorname{rot}\vec{H} = 0. \tag{4.5}$$

Newton's gravitation law with k as Newton's constant of gravitation is

$$\operatorname{div}\vec{g} = -4\pi k\mu.$$

Here, μ is the density of that mass distribution that causes the force of gravity $\vec{g}$. If one compares this with the third relation of (4.5), then $2\omega^2$ corresponds to a negative mass density

$$\mu = -\frac{\omega^2}{2\pi k}$$

acting repulsively. It is created by the centrifugal force. With a revolution time of 10 seconds, it corresponds to the mass density of a compact white dwarf. The third equation of (4.5) shows that the Coriolis field is the source of the centrifugal force.

We consider the equations of motion of a particle with the proper mass m_0, the relativistic mass $m = m_0 \big/ \sqrt{1-v^2}$, and the 4-velocity

$$w_m = \frac{1}{\sqrt{1-v^2}} \{-iv_\alpha, 1\}, \quad v^2 = v^\alpha v_\alpha . \tag{4.6}$$

If one exposes such a particle or a particle field to the rotating system, if one splits the equation of motion

$$w^n w_{m||n} = 0 \tag{4.7}$$

into space-like and time-like components and multiplies them by the proper mass m_0, and if one divides it by the Lorenz factor, one comes up with

$$\begin{aligned} (\vec{v}\,\mathrm{grad})\vec{p} + p^{\cdot} &= 2m\left(\vec{v} \times \vec{H}\right) + m\vec{F} \\ (\vec{v}\,\mathrm{grad})m + m^{\cdot} &= m\vec{F}\vec{v}, \quad \vec{p} = m\vec{v} \end{aligned}, \tag{4.8}$$

the effect of the forces on the particle.

Although the rigid rotator is not realized by Nature, this model can explain some physical effects emerging in rotating systems. We make some remarks concerning such rotating systems.

The optical experiment of Sagnac [6], the pedant to the Michelson experiment, apparently speaks in favor of the absoluteness of rotation: Rays of light orbiting in the opposite directions on a platform produce interferences if a mirror system is adjusted in such a way that the rays meet after a circulation. If the platform is set in rotation, the interference fringes are shifted. The shift is a function of the angular velocity of the platform. As the reason, one has accepted that the speeds of light are different in the opposite directions and, thus, refer to the absoluteness of the rotational motion. Only in the rest system do both light rays have the same speed.

This possible consequence of the Sagnac experiment is not only unsatisfactory, but it also contradicts the general relativity principles as well. Therefore, we looked for another interpretation of the shift of the fringes observed in the Sagnac interferometer: The experimental arrangement on the rotating platform can be considered to be in relative rest; however, gravitational energy orbits the platform. The arising forces lead to an extension and to a shortening of the optical paths and to changes in the physical flow of time. Light rays moving in opposite directions cover different optical distances, whereby the measurable fringe shifts emerge. However, they require a different time. The quotients of these distances, the velocities of the light rays, are equal.

Therefore, the constancy principle is also valid for accelerated reference systems, and one can show that the very gravitational forces affect Newton's bucket and the optical paths of the Sagnac experiment. We have shown this in detail in our paper [7] with the help of the object of anholonomy.

Corum [7] might have been the first to approach the problem. With strictly relativistic methods, he consequently used the anholonomic systems which we prefer. However, he did not mention the constancy principle although it directly follows from his ansatz. We made up this in one of our articles [8]. We do not follow Corum; he claimed a rotation-related change in the frequency of light. Frequency distortions would lead to disintegration of the fringe pattern on the interferometer.

In an early paper [9], we introduced a rotating observer into the Einstein cosmos. Evidently, this does not lead to a genuine rotating model, but it exhibits a structure for investigating the interplay between the energy content of matter and field. Moreover, in [3], we simplified the genuine Kerr metric

$$ds^2 = {dx^1}^2 + {dx^2}^2 + \left[\alpha dx^3 + i\alpha\omega\sigma dx^4\right]^2 + a_S^2\left[-i\alpha\omega\sigma dx^3 + \alpha dx^4\right]^2$$

to a model with induced rotation and free of gravitation,

$$ds^2 = {dx^1}^2 + {dx^2}^2 + \left[\alpha dx^3 + i\alpha\omega\sigma dx^4\right]^2 + \left[-i\alpha\omega\sigma dx^3 + \alpha dx^4\right]^2,$$

to better explain the rotational effects. The simplified Kerr metric describes a flat space, parametrized in elliptic-hyperbolic coordinates and endowed with rotation resorting to a differential rotation law. Although this model is more complicated than our ansatz (3.1) and (3.4), the same type of gravimagnetic equations (4.1) and (4.2) emerge as the consequence of anholonomy.

5. CONCLUSIONS

We transformed anholonomic structures written with coordinate methods into the tetrad way of writing and complemented this formalism with some formulae concerning the curvature of space. We added several examples where anholonomy plays an important role.

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