ARG-2017-01

The present paper is a translation of the talk 'Karl Schwarzschild and what happened with his work' at a meeting of the Leibniz Sozietät in Berlin on the occasion of the 100th anniversary of the death of Karl Schwarzschild in December 2016. Published in Leibniz Online 26 (2017).

KARL SCHWARZSCHILD AND WHAT HAPPENED WITH HIS WORK

Rainer Burghardt*

1. The gravitational model of Schwarzschild and attempts to generalize it

Soon after Einstein had published the first paper on the general theory of relativity, Karl Schwarzschild proposed two solutions of Einstein's field equations to describe the exterior and interior of a stellar object. It is certainly of interest to investigate how these solutions were dealt with in the course of a century and to discuss these results critically. First of all, it must be noted that the Schwarzschild line element of the exterior solution, which already contains the essentials of the theory, was cast by Hilbert into the form which we now call the standard Schwarzschild metric. The line element

$$ds^2 = \frac{1}{1-\frac{2M}{r}}dr^2 + r^2 d\vartheta^2 + r^2 \sin^2\vartheta d\varphi^2 - \left(1-\frac{2M}{r}\right)dt^2 \qquad (1.1)$$

describes the section of a curve in the curved space using polar coordinates, where M is the mass of the field-generating object in geometric units.

In 1916, Flamm put the model on a geometrical basis. The space part of the metric (1.1) can be interpreted as a fourth-order surface, which is embedded into a higher-dimensional flat space. This surface is called Flamm's paraboloid.

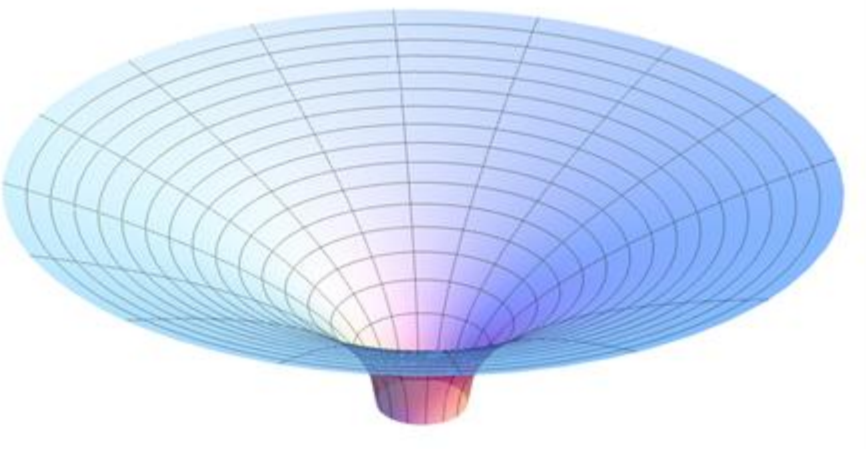

Fig. 1. Flamm's paraboloid

* e-mail: arg@aon.at, home page: http://arg.or.at/

The circle at the waist of this surface is located at $r = 2M$. It is called event horizon or Schwarzschild horizon. For $r < 2M$ the surface does not exist, Eq. (1.1) cannot describe this area. In (1.1) one can see that because of

$$r \to 2M, \quad \frac{1}{1-2M/r} \to \infty \tag{1.2}$$

the metric has a singularity at the event horizon. To this day it has been discussed whether this singularity is a real or a coordinate singularity. If the latter were true, the singularity would have to be corrected by a suitable coordinate choice. This is also the case if the Einstein-Rosen coordinates [1] are used. The radial curves on Flamm's paraboloid have the equation

$$R^2 = 8M(r-2M). \tag{1.3}$$

Using in (1.1) the extradimension R instead of the radial coordinate r one gets the singularity-free metric

$$ds^2 = \frac{R^2+16M^2}{16M^2}dR^2 + \left(\frac{R^2+16M^2}{8M}\right)^2\left(d\vartheta^2 + \sin^2\vartheta\, d\varphi^2\right) - \frac{R^2}{R^2+16M^2}dt^2 \ . \tag{1.4}$$

In this case is located the event horizon at $R = 0$. However, there can be no doubt that the physical parameters behave strangely at the event horizon. The speed of a free-falling object would reach the velocity of light at the event horizon, the gravity would be infinitely high at this location and the flow of time would stop. These are properties that a good physical theory should not have. We therefore assume that the event horizon cannot be reached and we will show why this is so.

We first deal with attempts to transform the metric (1.1) by means of a coordinate transformation in such a way that the region $r < 2M$ can also be included. Eddington [2] and later Finkelstein [3] have brought with the transformation

$$t = t' \pm 2M\ln(r-2M) \tag{1.5}$$

the metric into the form

$$ds^2 = dr^2 - dt^2 + v^2(dr+dt)^2 + r^2 d\vartheta^2 + r^2\sin^2\vartheta\, d\varphi^2, \quad v = -\sqrt{2M/r} \ . \tag{1.6}$$

It is singularity-free and superficially it appears that the radial coordinate r can now also run into the region $r < 2M$. If one processed the bracket in (1.6), one gets

$$ds^2 = \left(1+v^2\right)dr^2 + 2v^2 dr\,dt + r^2 d\vartheta^2 + r^2\sin^2\vartheta\, d\varphi^2 - (1-2M/r)dt^2 \tag{1.7}$$

and recognizes that a cross term occurs in the formula. This means that an oblique-angled coordinate system is used, which is unsuitable for the representation of physical variables. To avoid this deficiency, an orthogonal time coordinate must be introduced

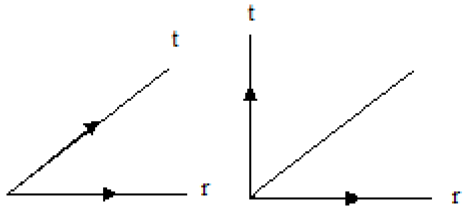

Fig. 2. Eddington-Finkelstein- coordinates

and thus is again there, where one was before: the standard Schwarzschild coordinate system.

Kruskal [4] has undertaken a further advance into the region beneath the event horizon. He introduced a coordinate system that is supposed to represent new regions of the Schwarzschild geometry that are not known yet. The Kruskal system contains four sectors, so we speak of the fourfold truth and of the maximum extension of Schwarzschild theory. In the Kruskal diagram the sector II is interpreted as a black hole absorbing matter; the sector IV is interpreted as a white hole that ejects matter. We will examine how the diagram comes about. We interpret the hyperbolae in the drawing as a sequence of Lorentz transformations from a static system to an accelerated system in pseudo-representation

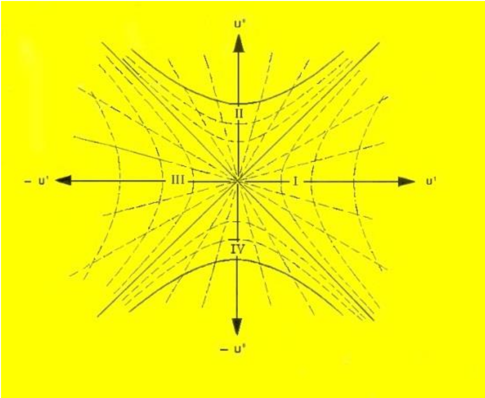

Fig. 3. Kruskal diagram

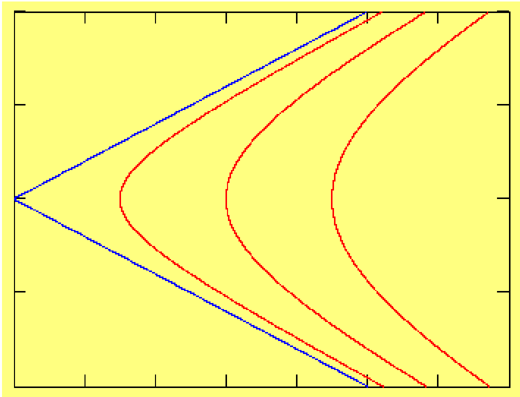
Fig. 4. Family of hyperbolae

$$Y^2 = \left(u^1\right)^2 - \left(u^0\right)^2 .$$

The hyperbolas are assigned to the values $Y = 0,1,2,3$ in Fig.4. Interpreted in this way, the Kruskal diagram is not a space-time diagram, but the representation of an accelerated motion. The two similar sectors I and III represent a bradyonic motion - an object is accelerated from the event horizon to infinity. Sectors II and IV describe a tachyonic motion, ie, a motion with superluminal speed - an object would have an infinitely high speed at the event horizon and slow down to infinity at the speed of light.

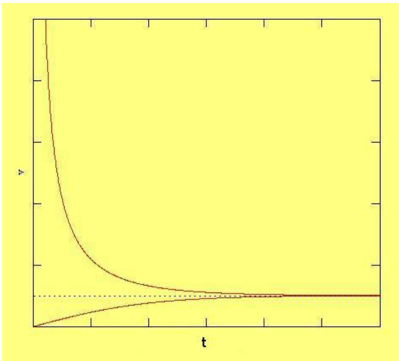

Fig. 5. Kruskal velocities

The first somewhat descriptive approach is completed by the following considerations: We derive the transformation matrix $x^{i'}{}_{|i}$ from the Kruskal coordinate system ($x^{1'} = u^1, \quad x^{4'} = iu^0$) by differentiation. From the Schwarzschild metric (1.1) and the Kruskal metric

$$ds^2 = \gamma^2\left(du^{1^2} + du^{4^2}\right) + r^2 d\vartheta^2 + r^2 \sin^2\vartheta\, d\varphi^2, \quad \gamma^2 = \frac{32M^3}{r} e^{-\frac{r}{2M}} \tag{1.8}$$

we read the 4-beine and with

$$L^{m'}_{m} = \overset{m'}{e}_{i'}\, x^{i'}{}_{|i}\, \underset{m}{e}^{i} \tag{1.9}$$

we obtain the Lorentz transformation

$$L_m^{m'} = \begin{pmatrix} \cos i\chi & & & -\sin i\chi \\ & 1 & & \\ & & 1 & \\ \sin i\chi & & & \cos i\chi \end{pmatrix} \tag{1.10}$$

which has the scaled coordinate time $\chi = t/4M$. From these results the Kruskal velocity for the bradyonic case

$$v_K = \text{th}\chi\,. \tag{1.11}$$

The gravitational field is described by the Ricci-rotation coefficients which transform inhomogeneously into a relatively moving system:

$$'A_{m'n'}{}^{s'} = L_{m'n's}^{m\,n\,s'} A_{mn}{}^{s} + L_{s}^{s'} L_{n'|m'}^{s}\,. \tag{1.12}$$

Inserting the Schwarzschild values into A and calculating the second term with (1.10), one obtains the Kruskal acceleration, which can also be observed from the static system. The effective Kruskal acceleration is

$$K_m^e = K_m + E_m, \quad K_m = \{1,0,0,0\}\frac{1}{4M\sqrt{1-2M/r}}\,, \tag{1.13}$$

wherein the actual Kruskal acceleration K is decreased by the (negative) gravitational acceleration E. From (1.12) it can be seen that the field quantities in the Kruskal system are only defined in the exterior Schwarzschild region, and Einstein's field equations cannot describe the interior Schwarzschild region. It is clear that the choice of a new coordinate system cannot alter the geometrical and physical content of a theory. This is the basic prerequisite for a physical theory.

A further advance into the interior region is achieved by the change of the Schwarzschild metric to

$$\frac{1}{1-\frac{2M}{r}}dr^2 \to -\frac{1}{\frac{2M}{r}-1}dr^2, \qquad -\left(1-\frac{2M}{r}\right)dt^2 \to \left(\frac{2M}{r}-1\right)dt^2\,.$$

In this notation it is possible that $r < 2M$. Then the metric has the form

$$ds^2 = -\frac{1}{\frac{2M}{r}-1}dr^2 + r^2 d\vartheta^2 + r^2\sin^2\vartheta\, d\varphi^2 + \left(\frac{2M}{r}-1\right)dt^2\,. \tag{1.14}$$

The red expression is now time-like, the blue expressions space-like. It is noticeable that the variable r occurs in both meanings, which is contradictory. Nevertheless, Eq. (1.14) is used as the basis for the definition of a black hole. A stellar object could collapse up to the event horizon and become a black hole. The event horizon would then be a one-way membrane, everything can fall in, and nothing can get out, not even light. We do not join this idea and stick to Laue: points with $r < 2M$ do not exist for the exterior Schwarzschild solution.

2. Free fall and black hole

Repeatedly attempts have been made to describe motions which are intended to allow the penetration of objects into the inner region of the exterior Schwarzschild model.

Gautreau [6-7] and Hoffmann [8] assume that bodies passing the event horizon continue their motion with superluminous velocity in the inner region r < 2M. De Sabbata, Pavšič, and Recami [9] examine in detail this tachyonic (motion with velocity higher than light) and bradyonic behavior (designating a motion with velocity lower than light).

Janis [11] introduces a new reference system for measuring the speed of a freely falling body and shows that in this system the speed is less than the speed of light at the event horizon r = 2M. Cavalleri and Spinelli [12,13] recognize as the cause a special co-ordinate choice and a misinterpretation of the particle speed. In an answer, Janis [14] tries to support his point of view by a comparison with the Minkowski space.

From Jaffe's and Shapiro's [15-18] computations it becomes clear that particles falling to a stellar object accelerate at first then, however, become slower again. Their transition speed at the event horizon would then be smaller than the speed of light. McGruder III [19] recognizes a repulsive effect in the environment of the event horizon. Baierlein [20] contradicts Jaffe and Shapiro and points out that the incorrect choice of the co-ordinate time in place of the physical time has led to these results.

Tereno [21] integrates the radial equation of motion in a likewise way and shows that radial geodesics do not become null-lines at the event horizon. This means that material particles can fall into a black hole with a velocity less than the velocity of light. Mitra [21,22] attributes this view to an error concerning the limit operation in Tereno's calculations[1]. In order to confirm his point of view Tereno performs some more calculations in Kruskal co-ordinates. Mitra answers with a detailed analysis in Kruskal co-ordinates. Since Tereno contradicts again, Mitra [23,24] consults the Shapiro-Teukolsky [25] [r,t]-relation and by the use of cycloidal co-ordinates arrives again at the result v = c at the event horizon.

Unaffected by this criticism Crawford and Tereno [26] returned to this problem in a later paper. After a detailed discussion on reference systems in the Schwarzschild field they specify a formula from which the speed is calculable in such a way that a freely falling body could enter a black hole. Krori and Paul [27], Lynden-Bell and Katz [28], and Salzmann and Salzmann [29] were concerned with similar problems. We refer to the papers of Logunov, Mestverishvili, and Kiselev [30] and Loinger [31]. De Sabbata and Shah [32] showed that the red shift at the event horizon becomes infinite. Another aspect has been considered by Royzen [33].

A method described by Misner, Thorne, and Wheeler [34] in their textbook seems to have been widely accepted. It was adopted in some textbooks and in hundreds of works. On several pages the authors deal with the free fall of an object which falls away from an arbitrary position to a central mass. They note that seen from an observer at infinity, the object asymptotically approaches the event horizon, while an accompanying observer has reached the position after a relatively short period of time. This is worse than with Schrödinger's cat. For one observer it lives forever, for the other it is quickly dead. The problem was represented by MTW in Fig. 6. We want to examine the method critically.

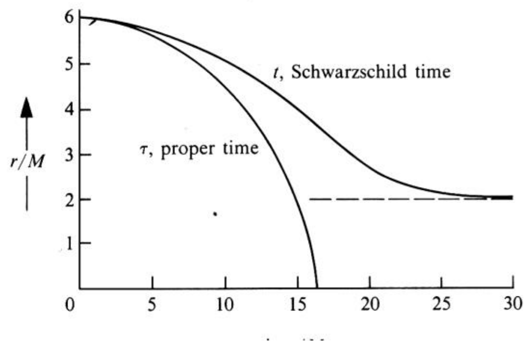

Fig. 6. The free fall from an arbitrary position by Misner, Thorne, and Wheeler

[1] After integration of the radial equation of motion Tereno arrived at an expression $\lim_{\varepsilon \to 0} \frac{\varepsilon}{\varepsilon}$ at the event horizon and implements the limit operation to this fraction, so that the indefinite form 0/0 develops. Due to this result he argues that the speed of a particle at the event horizon can take any value. However, since Bernoulli and de l'Hospital one knows that ε can be arbitrarily small, even infinitely small, but still not 0. Therefore, the fraction has to be reduced. It remains $\lim_{\varepsilon \to 0} 1 = 1$. At the event horizon one obtains ε = 0 and for the speed of light the value 1.

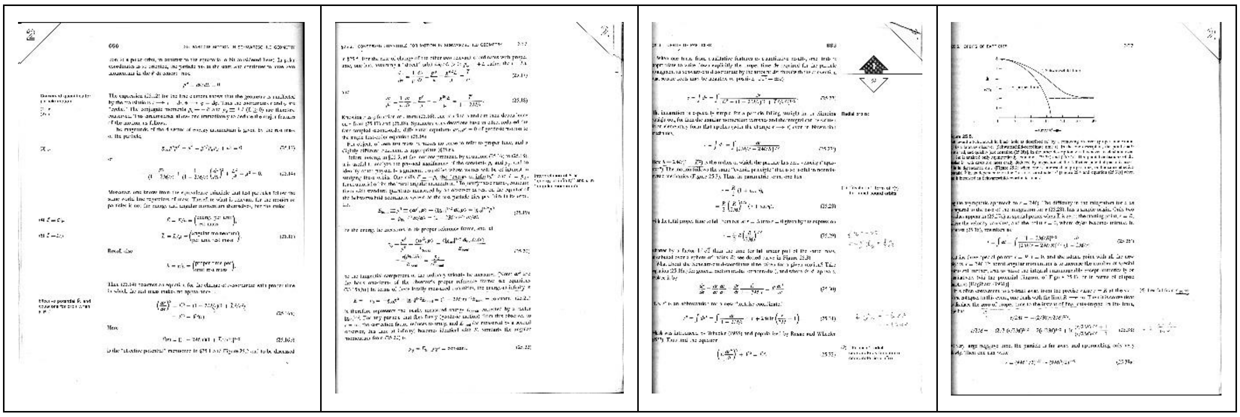

Fig. 7. The derivation by Misner, Thorne, and Wheeler

We simplify the derivative represented by MTW on several pages (Fig. 7) by removing variables which are not directly in use, and distance ourselves from the term 'energy at infinity' introduced by MTW. There remains a five-line, which leads to the results of MTW. From its simple structure it can be seen that the derivation is erroneous.

The 4-dimensional line element is split into a space-like part and a time-like part depending on the state of motion of an observer.

$$ds^2 = dx^2 - dT^2 \qquad = dx'^2 - dT'^2 \qquad = dx''^2 - dT''^2 \qquad (2.1)$$

In rest — Fall from an arbitrary position — Fall from the infinite

dx, dx', dx'' are the proper lengths measured by the observers, dT, dT', dT'' are their proper times. The error addressed is that the proper length measured by an observer who falls away from a finite position, has been combined by MTW with the proper time of an observer coming from the infinite, thus

$$ds^2 = dx'^2 - dT''^2 . \qquad (2.2)$$

If one corrects this approach, one comes to quite different results.

v' is the velocity of an observer coming from a finite position r_0, v'' the velocity of an observer coming from the infinite. v_0 is the velocity of an observer coming from the infinite at the moment when he has reached the position r_0 from which the observer of interest falls. Thus, the initially unknown quantity v' can be determined. One makes up the (relativistic) difference of the speed $v'' = -\sqrt{2M/r}$ known from the Schwarzschild theory and the velocity $v_0 = -\sqrt{2M/r_0}$ at the position r_0 of the reference observer which we have envisaged. The relation of the speeds is shown in Fig. 8.

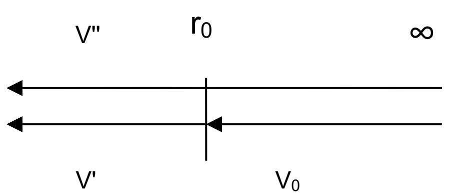

Fig. 8. The velocities

MTW

$$ds^2 = dx'^2 - dT''^2$$

$$v' = \sqrt{v''^2 - v_0^2}, \quad \alpha' = \frac{\alpha''}{\alpha_0}$$

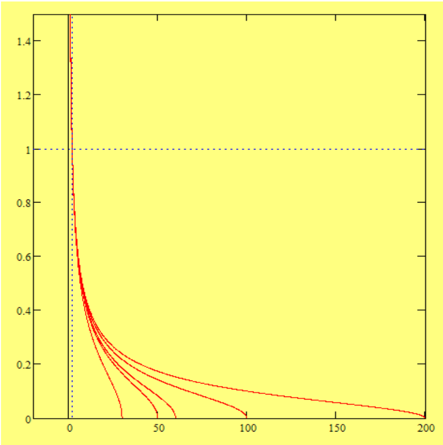

Fig. 9. Fall velocity from any position

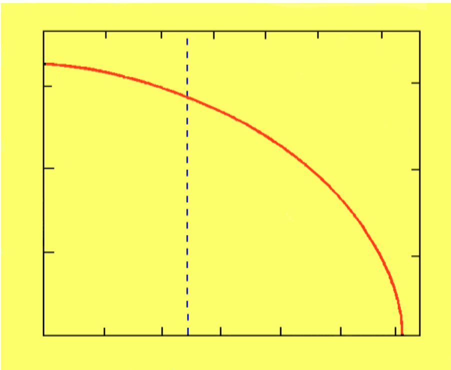

Fig. 11. Fall time

Strictly relativistic

$$ds^2 = dx'^2 - dT'^2$$

$$v' = \frac{v'' - v_0}{1 - v'' v_0}, \quad \alpha' = \alpha'' \alpha_0 \left(1 - v'' v_0\right)$$

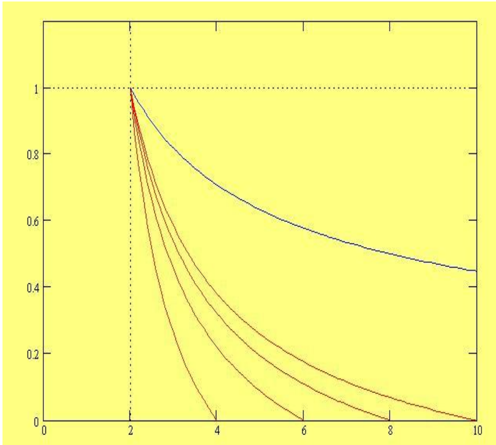

Fig.10. Fall velocity from any position

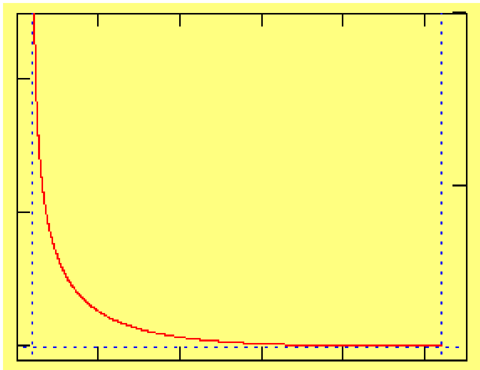

Fig. 12. Fall time

It can be seen that the fall velocity would also reach the speed of the light, whereby in the case of MTW it is possible to exceed this speed and the event horizon can be crossed. The course of the velocity in the case of MTW is somewhat meandering and physically unconvincing. In the strictly relativistic case, things are different: the speed of the observer who is falling down is

$$v' = \frac{dx}{dT}, \quad \frac{dt}{dT'} = \alpha', \quad \frac{\alpha dr}{dT'} = \alpha' v'. \tag{2.3}$$

The α are the Lorentz factors assigned to the velocities. If one integrates

$$dT' = \frac{\alpha''}{\alpha' v'} dr = \frac{1}{\alpha_0 \left(v'' - v_0\right)} dr \ , \tag{2.4}$$

one reads from the progression of the integral curve that the time needed for the progress of the object becomes asymptotically infinite in approximation at the event horizon $r = 2M$. No object can ever reach or even penetrate the event horizon (Fig. 12). It can be assumed that all attempts have failed to advance into the inner region of the exterior Schwarzschild solution. Posthumously on has laid hands on Schwarzschild.

3. Gravitational collapse

An important role plays the Schwarzschild theory in the attempt to describe the collapse of a star. A collapsing non-rotating star is supposed to be surrounded by a Schwarzschild field. The assumption is justified, because the Schwarzschild model has been well proved. The light deflection and the measured perihelion movement of Mercury are in full agreement with the theoretical predictions.

For the first time, a star collapse was described by Oppenheimer and Snyder [35]. The authors access the mathematical framework of an expanding cosmos of Tolman and adapt it to the collapse. The work is called the work that has justified the theory of black holes, even though the term 'black hole' was introduced later. It may be interesting for the scientific historians that no one has ever dealt in detail with this often-cited work.

However, a careful analysis of the OS model shows some inconsistencies. The collapsing star consists of incoherent dust without inner pressure and collapses in free fall. Before collapsing, it was infinitely large and filled an infinitely large universe completely. During the collapse its mass density grows. However, after a finite time it shrinks to a singular point.

When the surface of the star passes the event horizon, according to the OS no light can get away from this star, it becomes dark. Mitra [36] has shown that OS did not only forget a factor of 1/4 in the calculation of this effect, but the relevant quantity also has the wrong sign. In addition to the shortcomings described above, another deficit emerges: At the boundary surface, the interior and the exterior solutions do not match. The interior solution of the Einstein field equations, which describes the star, must be adapted to the exterior solution, which describes the surrounding gravitational field. The surfaces of the two solutions must touch each other and must have common tangents, ie the metrics and their first derivatives must coincide at the boundary of the two geometries. The first condition is fulfilled by OS, the second is not, as Nariai [37-39] has already recognized. The situation is illustrated in Fig 13.

Fig. 13. The junction condition of the OS geometries

From the foregoing it is clear that the OS model cannot describe a collapse and is not suitable for explaining a black hole.

Another model comes from McVittie [40]. It was later rediscovered by Weinberg and described in his textbook [41]. This model is based on two velocity definitions. These velocities do not combine according to Einstein's law of velocities. On the surface of the star the collapse velocity is the expression

$$v_{col} = -\frac{\sqrt{\frac{2M}{r_g} - \frac{2M}{r'_g}}}{\sqrt{1 - \frac{2M}{r'_g}}} \, . \tag{3.1}$$

This results in a velocity development as we know it from Fig 9. Both authors demand that their collapsing solutions match the exterior Schwarzschild field, but they have difficulties to organize this correctly. Again the surface of the star can pass through the event horizon and its destiny ends in a singularity.

In addition to these classical models, there are numerous approaches to the collapse, most of them are not exact solutions to Einstein's field equations. If one wants to take into account the pressure inside the star, one has one more variable, but not enough equations to determine it. Even if the model is not fully determinable, one is already satisfied if at least one can make statements about the evolution of the collapse. In addition, adjustments have been made with computer techniques.

It is common to all these approaches that a 'collapsing metric' is to be found by solving Einstein's field equations. However, it is precisely this strategy that is responsible for not achieving a satisfactory result. To search for a collapsing metric is to search for a line on a surface whose element describes not only the properties of the surface itself but also its change.

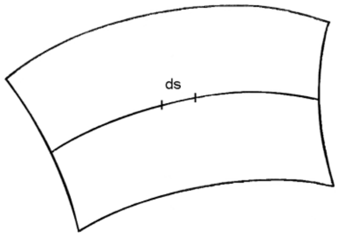

Fig. 14. The 'collapsing surface'

The logical content of this project is reminiscent of the story of the Baron Münchausen pulling himself out of the swamp by his own pigtail.

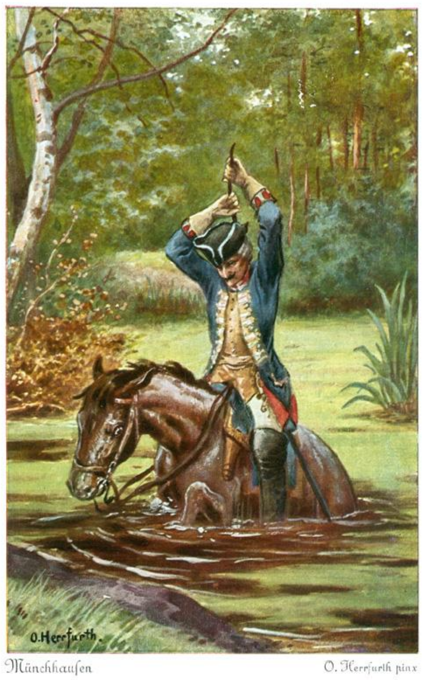

Fig. 15. Baron Münchhausen

If a model for a stellar collapse is to be successfully established, one must give up the idea of obtaining a metric as a solution to Einstein's field equations, which has all the geometrical properties that describe this collapse. Furthermore, the question arises why one has tried to supplement the exterior Schwarzschild solution with new models and why it has never been attempted to expand the interior Schwarzschild solution in such a way to allow a collapse. We [42] have successfully attempted the latter approach.

4. The inner Schwarzschild solution and the collapse

We start with the line element of the static interior Schwarzschild solution

$$ds^2 = \mathcal{R}^2 d\eta^2 + \mathcal{R}^2 \sin^2\eta \, d\vartheta^2 + \mathcal{R}^2 \sin^2\eta \sin^2\vartheta \, d\varphi^2 + \left[3\mathcal{R}\cos\eta_g - \mathcal{R}\cos\eta\right]^2 di\psi^2 . \qquad (4.1)$$

The space-like part of the metric describes the line element of a 3-dimensional spherical cap on a hyper-sphere with the radius $\mathcal{R}$. The latter is chosen in such a way that this surface has common tangents (cutting tangents) with Flamm's paraboloid. In the time-like part of the metric $\mathcal{R}\, di\psi = i dt$ is the geometric expression for the time. If one wants to derive a collapsing model from the static model, the radius of the hyper-sphere must be time-dependent

$$\mathcal{R} = \mathcal{R}(t) . \qquad (4.2)$$

If the star collapses the hyper sphere shrinks, ie it slides down Flamm's paraboloid and its radius decreases.

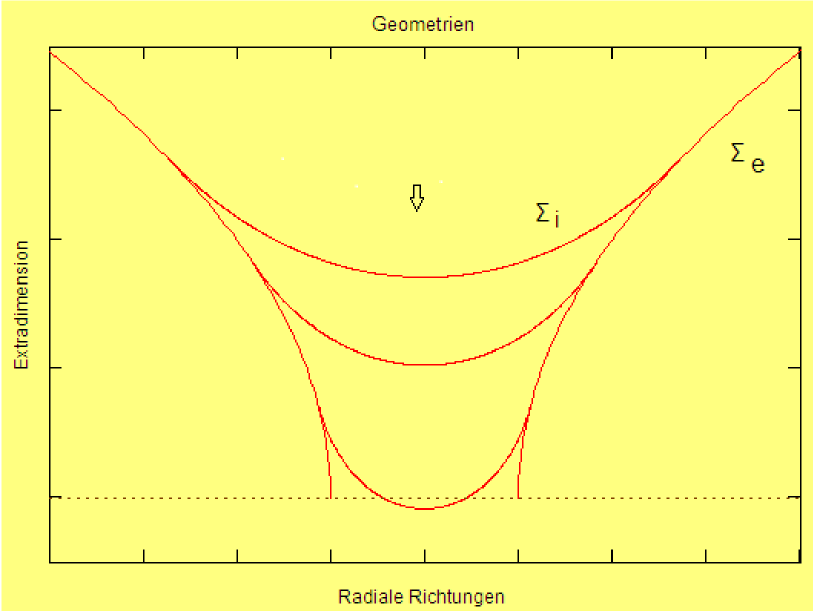

Fig. 16. The collapsing interior solution

It can be seen from the drawing that a collapsing surface does not exist, but a family of surfaces which are snapshots of the static interior solution, snapshots which cannot describe the collapse. Accordingly, no coordinate system exists which covers a collapsing surface and comoves with it. The star is surrounded by the exterior Schwarzschild field that is represented geometrically by Flamm's paraboloid. The latter remains unchanged during the collapse. This corresponds to Birkhoff's theorem: The collapse has no influence on the exterior field. However, Flamm's paraboloid determines the development of the collapse. The spherical cap slides down Flamm's paraboloid, its radius of curvature is always half as long as the radius of curvature of the Schwarzschild parabola at the boundary surface and is thus determined at any time. Here it is clear that the manner in which interior geometry has to behave is not a property of this surface, but is communicated to it by the exterior geometry. At any time of the collapse, the exterior geometry determines the curvature of the interior geometry. Models that want to implement the properties of the collapse in their own metric will be unsuccessful. Besides the question arises of why new solutions are sought although interior and exterior Schwarzschild solutions form a unity and can also be represented by a common mathematical calculus [43].

The stress-energy-momentum tensor of the interior solution is simply established

$$T_{mn} = \begin{pmatrix} -p & & & \\ & -p & & \\ & & -p & \\ & & & \mu_0 \end{pmatrix} . \tag{4.3}$$

The stellar object is approximately represented by a homogeneous fluid sphere. The mass density

$$\kappa\mu_0 = \frac{3}{\mathcal{R}^2} \tag{4.4}$$

increases if the radius of the sphere and thus the volume of the star shrinks. The pressure

$$\kappa p = -\frac{3}{\mathcal{R}^2}\frac{\cos\eta_g - \cos\eta}{3\cos\eta_g - \cos\eta} \tag{4.5}$$

is negative. η is the polar angle to the cap of the sphere in the embedding space. η_g is the polar angle at the boundary surface, thus the aperture angle of the spherical cap. It is immediately apparent that the pressure at the boundary is expected to be zero, but increases inwards. At the center of the star ($\eta = 0$) one has

$$\kappa p_c = \frac{3}{\mathcal{R}^2}\frac{1-\cos\eta_g}{3\cos\eta_g - 1}. \tag{4.6}$$

For $3\cos\eta_g = 1$ the pressure at the center of the star is infinite. If one converts the aperture angle to the position r_g of the boundary, one obtains as a parameter for the minimum size of a star with the mass M

$$r_H = 2.25M, \tag{4.7}$$

a value that is higher than the event horizon of the exterior solution ($r = 2M$). In the framework of Schwarzschild theory, a star cannot be arbitrarily small. Its volume always covers the hypothetical event horizon. All reasoning about what might happen at it and what it means mathematically is therefore superfluous. We call r_H the *pressure horizon* of the star. If a tube is drilled through the center of the star, and a body is dropped into the star, the latter emerges again on the opposite side, falls back, and oscillates back and forth. If the star is sufficiently small, the body reaches the speed of light as it passes through the center [43]. Thus, there exists a *velocity horizon* which is identical with the pressure horizon. This also shows that stellar objects can only collapse to a minimum size in a theory of collapsing stars which is based on the interior Schwarzschild solution. This is determined by the *inner horizon*. As a result of this, the term 'black hole' has no right to exist within the framework of the complete Schwarzschild theory. In the following we will show that this is the case.

.

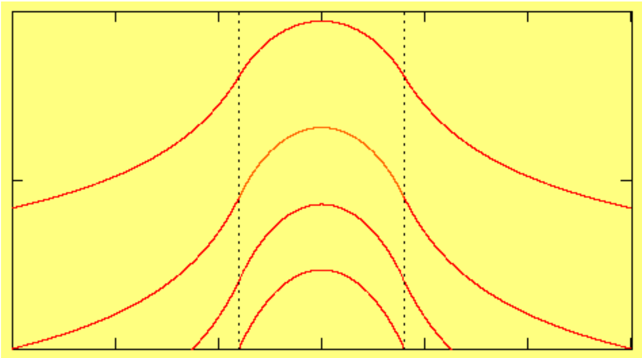

Fig. 17. The velocity horizon

We note a relation known from the static model and supplement it with an analogous one for the comoving system:

$$\frac{1}{r}r_{|1} = \frac{a_R}{r}, \quad r_{|4} = 0, \quad \frac{1}{r'}r'_{|1'} = \frac{a_I}{r}, \quad r'_{|4'} = 0. \tag{4.8}$$

The auxiliary variable r' with the value range $[0,\ldots,r'_g]$ is referred to in literature as a comoving radial coordinate. r'_g is the value of r'_g at the surface of the star. However, we do not make use of this interpretation since we cannot use or are not able to use a coordinate system for the collapsing model. We have explained this in detail above.

The following relations apply:

$$\begin{aligned} a_R^2 &= 1 - \frac{r^2}{\mathcal{R}^2}, \quad v_R = -\frac{r}{\mathcal{R}}, \quad \mathcal{R} = \sqrt{\frac{r_g^3}{2M}} \\ a_I^2 &= 1 - \frac{r'^2}{\mathcal{R}_0^2}, \quad v_I = -\frac{r'}{\mathcal{R}_0}, \quad \mathcal{R}_0 = \sqrt{\frac{r'^3_g}{2M}} \end{aligned}. \tag{4.9}$$

At the beginning of the collapse is $r_g = r'_g$ and $\mathcal{R} = \mathcal{R}_0$. Furthermore, we require the two velocities defined in (4.9) to be composed to the velocity of collapse according to Einstein's law of addition of velocities

$$v_C = \frac{v_R - v_I}{1 - v_R v_I}. \tag{4.10}$$

Together with (4.2), we have set up the collapsing Schwarzschild model. It has only to be shown that with this approach the field equations for both the comoving and the non-comoving observer systems are fulfilled.

Furthermore, the conservation law should now be satisfied with the time-dependent variables p and μ_0. We will not go into details about it, but we only notice that Einstein's field equations

$$R_{mn} = -\left[U^s{}_{\underset{1}{\|}s} + U^s U_s\right] h_{mn}$$
$$-\left[B_{n\underset{2}{\|}m} + B_n B_m\right] - b_n b_m \left[B^s{}_{\underset{2}{\|}s} + B^s B_s\right]$$
$$-\left[C_{n\underset{3}{\|}m} + C_n C_m\right] - c_n c_m \left[C^s{}_{\underset{3}{\|}s} + C^s C_s\right]$$
$$-\frac{1}{2}R = \left[U^s{}_{\underset{1}{\|}s} + U^s U_s\right] + \left[B^s{}_{\underset{2}{\|}s} + B^s B_s\right] + \left[C^s{}_{\underset{3}{\|}s} + C^s C_s\right]$$

have the same form in the comoving system as in the static system, but the field quantities are time-dependent and have a fourth time-dependent component. Furthermore, the static model has a new quantity

$$\mathcal{F}_m = \frac{1}{\mathcal{R}} \mathcal{R}_{|m} \rightarrow E_m^{kol} \tag{4.11}$$

which describes the evolution of the collapse. In the non-comoving system, the effect of the collapse is most easily seen. The total attraction force is composed of the gravitational acceleration which is already present in the static model and an additional acceleration which originates from the collapse and which contains the quantity (4.11)

$$E_m^{tot} = E_m^{grav} + E_m^{kol} . \tag{4.12}$$

Despite the complicated formal structure the results,1 are quite simple and easy to understand.

The most important property of the model is obtained from the integration of the collapse velocity. The star shrinks along the radial direction

$$v_C = \frac{dx^1}{dT}, \quad dx^1 = \alpha_R dr, \quad \frac{dT}{dT'} = \alpha_C .$$

Thus, using a Lorentz relation one has

$$\frac{\alpha_R dr}{dT'} = \alpha_C v_C = \alpha_R \alpha_I (v_R - v_I)$$

and

$$dT' = \frac{1}{\alpha_I (v_R - v_I)} dr . \tag{4.13}$$

Since the quantities α_I and v_I are constant at the surface of the star ($r = r_g, r_g' = \text{const.}$), the expression can easily be integrated in the interval $[r_H, r_g']$. The result is plotted in Fig. 18.

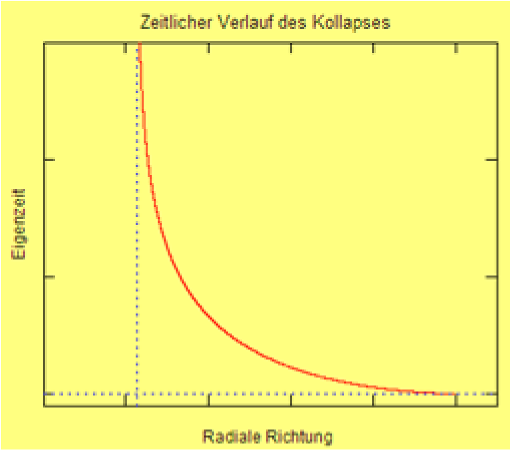

Fig. 18. The progress in time of the collapse

The result confirms our expectations. The surface of the star shrinks, but can reach the inner horizon (dashed line) only asymptotically, ie only after an infinitely long time. The full Schwarzschild model does not allow black holes. During the collapse, an ECO (Eternally Collapsing Object) is generated. The existence of such a stellar object was predicted by the Indian physicist Mitra [44] on the basis of astrophysical considerations. Once the star has collapsed close to the inner horizon, its external characteristics hardly differ from what is attributed to a hypothetical black hole.

All this could have already been seen, if one earlier has studied Schwarzschild's paper of 1916 and not just 100 years later, in the current year 2016.

We recapitulate the characteristics of the complete Schwarzschild solution:

- A stellar object with the two parameters p and μ_0 has an exterior field which is described by the exterior Schwarzschild solution, its interior by the interior Schwarzschild solution.
- The gravitational force inside the star is regular everywhere, in the center of the star it is zero.
- The interior solution always covers the event horizon of the exterior solution, so that the event horizon never can be experienced. Thus, all considerations which refer to an exotic situation at this location are obsolete.
- In particular, all attempts to extend the exterior Schwarzschild solution beneath the event horizon by a new choice of the coordinates or to allow a motion into this region are excluded by the structure of the model.
- Stars can collapse to an ultra-massive object. They can never shrink to a point singularity with infinitely high space curvature and mass density.

There are no unpleasant peculiarities in the complete Schwarzschild model. It satisfies all the prerequisites that one desires from a field theory.

That's just Schwarzschild!

[1] Einstein A., Rosen N., *The particle problem in the general theory of relativity*. Phys. Rev. **48**, 73, 1935

[2] Finkelstein D., *Past future asymmetry of the gravitational field of a point particle*. Phys. Rev. **110**, 965, 1958

[3] Eddington A. S., *A comparison with Whitehead's and Einstein's formulas*. Nature **113**, 192, 1924

[4] Kruskal M. D., *Maximal extension of the Schwarzschild metric*. Phys. Rev. **119**, 1743, 1960

[5] Gautreau R., *Alternative views to black holes.* Report New Jersey Institute of Technology 1980

[6] Gautreau R., *On Kruskal-Novikov co-ordinate systems.* Nuov. Cim. **56 B**, 49, 1980

[7] Gautreau R., *On the light cone inside the Schwarzschild radius*. Report: New Jersey Institute of Technology

[8] Gautreau R., Hoffmann R. B., T*he Schwarzschild radial coordinate as a measure of proper distance.* Phys. Rev. **D 17**, 2552, 1978

[9] De Sabbata V., Pavšič M., Recami E., *Black holes and tachyons*. Lett. Nuovo Cim. **19**, 441, 1977

[10] De Sabbata V., *Black holes and motion of test particles*. Tachyons, Monopoles and Rel. Topics 1978

[11] Janis A. I., *Note on motion in the Schwarzschild field*. Phys. Rev. **D 8**, 2366, 1973

[12] Cavalleri G., Spinelli G., *Note on the motion in Schwarzschild field*. Phys. Rev. **D 15**, 3065, 1977

[13] Cavalleri G., Spinelli G., *Motion of particles entering a Schwarzschild field.* Lett. Nuovo Cim. **6**, 5, 197

[14] Janis A. I., *Motion in the Schwarzschild field. A reply.* Phys. Rev. **D 15**, 3068, 1977

[15] Jaffe J., Shapiro I. I., *Comment on the definition of particle velocity in a Schwarzschild field.* Phys. Rev. **D 8**, 4642, 1973

[16] Jaffe J., *The escape of light from within a massive object.* Mon. Not. Roy. Astr. Soc. **149**, 395, 197

[17] Jaffe J., Shapiro I. I., *Lightlike behavior of particles in a Schwarzschild field.* Phys. Rev. **D 6**, 405, 1972

[18] Jaffe J., *Collapsing objects and the backward emission of the light.*

Ann. Phys. **55**, 374, 1969
[19] McGruder III C. H., *Gravitational repulsion in the Schwarzschild field.* Phys. Rev. **D 25**, 3191, 1982
[20] Baierlein R., *Comment on the Jaffe-Shapiro definition of velocity.* Phys. Rev. **D 8**, 4639, 1973
[21] Tereno I., *Velocity at the Schwarzschild horizon revisited*. astro-ph/9905144
[21] Mitra A., *Comment on 'Velocity at the Schwarzschild horizon revisited' by I. Tereno*. astro-ph/9905175
[22] Mitra A., *Kruskal coordinates and mass of Schwarzschild Black Holes.* astro-ph/9904162
[23] Tereno I*., Another view on the velocity at the Schwarzschild horizon*. astro-ph/9905298
[23] Mitra A., *Comments on 'Another view on the velocity at the Schwarzschild horizon'*, astro-ph/9905329
[24] Mitra A., *Kruskal dynamics for radial coordinates. I.* gr-qc/9909062
[25] Shapiro S. L., Teukolsky S. A., *Black Holes, White Dwarfs, and Neutron Stars*. Wiley, New York 1983
[26] Crawford P., Tereno I., *General observers and velocity measurements in General Relativity.* gr-qc/0111073
[27] Krori K. D., Paul, B. B*., Light-like motions of particles in gravitational fields*. J. Phys. **A 10**, 1887, 1977
[28] Lynden-Bell D., Katz J., *Geometric extension through Schwarzschild r = 0*. Mon. Not. Roy. Astr. Soc**. 247**, 651, 1999
[29] Salzmann F., Salzmann G., *Acceleration of material particles to the speed of light in general relativity.* Lett. Nuovo Cim. **1**, 859, 1969
[30] Logunov A. A., Mestverishvili M. A., Kiselev V. V., *Black holes: a prediction of theory or phantasy?* gr-qc/041205
[31] Loinger A., *The black holes are fictive objects*. Preprint 2004
[32] De Sabbata V., Shah K. T., *Is there any gravitational field outside a black hole?* Nuovo Cim. **23**, 409, 1978
[33] Royzen I. I., *QUC against black holes?* gr-qc/0906.1929
[34] Misner C. W., Thorne K. S., Wheeler J. A., *Gravitation*. San Francisco 1973
[35] Oppenheimer J. R., Snyder H., *On continued gravitational contraction*. Phys. Rev. **56**, 455, 1939
[36] Mitra A., *The fallacy of Oppenheimer-Snyder collapse: No general relativistic collapse at all, no black hole, no physical singularity.* Astrophys. Sp. Sci. **332**, 432010, 2010
[37] Nariai H., *On the boundary conditions in general relativity.* Prog. Theor. Phys. **34**, 173, 1965
[38] Nariai, H., Tomita, K., *On the problem of gravitational collapse.* (1965) Prog. Theor. Phys. **34**, 155-172.
[39] Nariai H., Tomita K., *On the applicability of a dust-like model to a collapsing or anti-collapsing star at high temperature.* Prog. Theor. Phys. **35**, 777, 1966
[40] McViitie G. C*., Gravitational collapse to a small volume*. Astrophys. J. **140**, 401, 1964
[41] Weinberg S., *Gravitation and Cosmology.* John Wiley & Sons, New York 1972
[42] Burghardt R., *Collapsing Schwarzschild Interior Solution*. Journal of Modern Physics, **06**, 1895-1907, 2015, <http://dx.doi.org/10.4236/jmp.2015.613195>
[43] Burghardt R., *Raumkrümmung.* http://arg.boxes.at/Mono/Mono.pdf
Burghardt R., *Spacetime curvature.* http://arg.boxes.at/EMono/EMono.pdf
[44] Mitra A., *Black holes or eternally collapsing objects: A review of 90 years of misconception*. Foc. of Black Hole research. (astro-ph/0409049)