

Hilfdatei für 'Collapsing Schwarzschild interior'

Hilfsmetrik

#1.....

$$a_R = \sqrt{1 - \frac{r^2}{R_g^2}}, \quad v_R = -\frac{r}{R_g}, \quad R_g = \sqrt{\frac{r_g^3}{2M}}, \quad \alpha_R = \frac{1}{a_R}, \quad \partial_1 = \frac{\partial}{\alpha_R \partial r} = a_R \frac{\partial}{\partial r}$$

$$\frac{1}{a_R} a_{R|1} = \frac{1}{2a_R^2} (-v_R^2)_{|1} = -\alpha_R^2 v_R \left(-\frac{r}{R_g} \right)_{|1} = \alpha_R v_R \frac{1}{R_g} = \hat{U}_1$$

#2.....

$$B_1 = -\hat{e}_2 e_{2|1}^2 = \frac{1}{r} r_{|1} = \frac{a_R}{r}, \quad C_2 = -\hat{e}_3 e_{3|2}^3 = \frac{1}{r \sin \vartheta} \frac{\partial}{\partial \vartheta} (r \sin \vartheta) = \frac{1}{r} \cot \vartheta$$

#3.....

$$\hat{A}_{41}^4 = -\hat{e}_4 e_{4|1}^4 = -a_R \left(\frac{1}{a_R} \right)_{|1} = \frac{1}{a_R} a_{R|1} = \hat{U}_1$$

#4.....

$$B_{1|1} = \left(\frac{a_R}{r} \right)_{|1} = -\frac{a_R}{r^2} r_{|1} + B_1 \frac{1}{a_R} a_{R|1} = B_1 \hat{U}_1 \quad \text{mit \#2,3}$$

$$B_{1|1} + B_1 B_1 = B_1 \alpha_R v_R \frac{1}{R_g} = -\frac{1}{R_g^2}$$

#5.....

$$C_{2|2} = \left(\frac{1}{r} \cot \vartheta \right)_{|2} = \frac{1}{r^2} \frac{\partial}{\partial \vartheta} \cot \vartheta = -\frac{1}{r^2} (1 + \cot^2 \vartheta), \quad C_{2|2} + C_2 C_2 = -\frac{1}{r^2}$$

$$C_{2||2}^3 + C_2 C_2 = C_{2|2} - B_{22}^1 C_1 + C_2 C_2 - \frac{1}{r^2} + \frac{a_R^2}{r^2} = -\frac{v_R^2}{r^2} = -\frac{1}{R_g^2}$$

#6.....

$$\hat{U}_{|s}^s = \left(\alpha_R v_R \frac{1}{R_g} \right)_{|1} = \alpha_R^3 v_{R|1} \frac{1}{R_g} = -\alpha_R^2 \frac{1}{R_g^2}, \quad \hat{U}_{|s}^s + \hat{U}^s \hat{U}_s = (\alpha_R^2 v_R^2 - \alpha_R^2) \frac{1}{R_g^2} = -\frac{1}{R_g^2}$$

#7.....

$$\begin{aligned}
R_{mn} &= A_{mn}{}^s{}_s - A_{n|m} - A_{rm}{}^s A_{sn}{}^r + A_{mn}{}^s A_s, \quad A_s = A_{rs}{}^r \\
&= -B_{mn}{}^s{}_s - C_{mn}{}^s{}_s - \hat{U}_{mn}{}^s{}_s \\
&\quad - B_{n|m} - C_{n|m} - \hat{U}_{n|m} \\
&\quad - [B_{rm}{}^s + C_{rm}{}^s + \hat{U}_{rm}{}^s] [B_{sn}{}^r + C_{sn}{}^r + \hat{U}_{sn}{}^r] \\
&\quad + [B_{mn}{}^s + C_{mn}{}^s + \hat{U}_{mn}{}^s] [B_s + C_s + \hat{U}_s]
\end{aligned}$$

$$I = -[b_m b_n B_{|s}^s + c_m c_n C_{|s}^s + u_m u_n \hat{U}_{|s}^s]$$

$$\begin{aligned}
III &= -[B_{rm}{}^s B_{sn}{}^r + C_{rm}{}^s B_{sn}{}^r + \hat{U}_{rm}{}^s B_{sn}{}^r] \\
&\quad - [B_{rm}{}^s C_{sn}{}^r + C_{rm}{}^s C_{sn}{}^r + \hat{U}_{rm}{}^s C_{sn}{}^r] \\
&\quad - [B_{rm}{}^s \hat{U}_{sn}{}^r + C_{rm}{}^s \hat{U}_{sn}{}^r + \hat{U}_{rm}{}^s \hat{U}_{sn}{}^r] \\
&= -[B_m B_n + C_m C_n + \hat{U}_m \hat{U}_n]
\end{aligned}$$

$$\begin{aligned}
IV &= -b_m b_n B^s B_s - c_m c_n B_s C^s - \hat{U}_{mn}{}^s B_s \\
&\quad + B_{mn}{}^s C_s - c_m c_n C_s C^s - \hat{U}_{mn}{}^s C_s \\
&\quad - b_m b_n \hat{U}_s B^s - c_m c_n \hat{U}_s C^s - u_m u_n \hat{U}_s \hat{U}^s
\end{aligned}$$

$$\begin{aligned}
R_{mn} &= -[\hat{U}_{n|m} + \hat{U}_n \hat{U}_m] - u_m u_n [\hat{U}_{|s}^s + \hat{U}^s \hat{U}_s] \\
&\quad - [B_{n|m} - \hat{U}_{mn}{}^s B_s + B_n B_m] - b_m b_n [B_{|s}^s + \hat{U}_s B^s + B^s B_s] \\
&\quad - [C_{n|m} - \hat{U}_{mn}{}^s C_s - B_{mn}{}^s C_s + C_n C_m] - c_m c_n [C_{|s}^s + \hat{U}_s C^s + B_s C^s + C^s C_s]
\end{aligned}$$

$$\begin{aligned}
R_{mn} &= -[\hat{U}_{|s}^s + \hat{U}^s \hat{U}_s] h_{mn} \\
&\quad - [B_{n||m} + B_n B_m] - b_m b_n [B_{||s}^s + B^s B_s] \\
&\quad - [C_{n||m} + C_n C_m] - c_m c_n [C_{||s}^s + C^s C_s]
\end{aligned}$$

$$h_{mn} = \begin{pmatrix} 1 & & & \\ & 0 & & \\ & & 0 & \\ & & & 1 \end{pmatrix}$$

Schwarzschild innere Metrik

#8.....

$$U_1 = p\hat{U}_1, \quad p_{11} = (1-p)U_1$$

$$p[\hat{U}_{11} + \hat{U}_1\hat{U}_1] = U_{11} - \hat{U}_1 p_{11} + \hat{U}_1 U_1 = U_{11} - \hat{U}_1(1-p)p\hat{U}_1 + U_1\hat{U}_1 = U_{11} + U_1 U_1$$

$$U_{11} + U_1 U_1 = p[\hat{U}_{11} + \hat{U}_1\hat{U}_1]$$

#9.....

$$B_{n||m} + B_n B_m = B_{n|m} - U_{mn}{}^s B_s + B_n B_m$$

$$B_{4||4} + B_4 B_4 = -U_{44}{}^1 B_1 = U_1 B_1 = -\frac{p}{R_g^2}$$

$$B_{n||m} + B_n B_m = -\begin{pmatrix} 1 & & & \\ & 0 & & \\ & & 0 & \\ & & & p \end{pmatrix} \frac{1}{R_g^2}$$

Kollabierendes Modell

#10.....

$$v_{||1'} = -\left(\frac{r'}{R_0}\right)_{1'} = v_1 \frac{1}{r'} r'_{1'} = a_1 v_1 \frac{1}{r}, \quad v_{||m'} = \{1, 0, 0, 0\} a_1 v_1 \frac{1}{r}, \quad v_{||m} = \{\alpha_C, 0, 0, i\alpha_C v_C\} a_1 v_1 \frac{1}{r}$$

#11.....

$$\frac{1}{R_g} R_{g|m} = \frac{1}{R_g} \frac{1}{\sqrt{2M}} (r_g^{3/2})_{|m} = \frac{3}{2} \frac{1}{r_g} r_{g|m}$$

$$F_m = \frac{1}{R_g} R_{g|m} = \frac{3}{2} \frac{1}{r_g} r_{g|m}$$

#12.....

$$L_1^1 = \alpha_C, \quad L_1^4 = i\alpha_C v_C, \quad L_4^1 = -i\alpha_C v_C, \quad L_4^4 = \alpha_C$$

$$v_C = \frac{v_R - v_I}{1 - v_R v_I}, \quad v_R = \frac{v_C + v_I}{1 + v_C v_I}, \quad v_I = \frac{v_R - v_C}{1 - v_R v_C}$$

$$\alpha_C = \alpha_R \alpha_I (1 - v_R v_I), \quad \alpha_R = \alpha_C \alpha_I (1 + v_C v_I), \quad \alpha_I = \alpha_C \alpha_R (1 - v_C v_R)$$

$$\alpha_C v_C = \alpha_R \alpha_I (v_R - v_I), \quad \alpha_R v_R = \alpha_C \alpha_I (v_C + v_I), \quad \alpha_I v_I = \alpha_R \alpha_C (v_R - v_C)$$

#13.....

$$\hat{U}_{1'} = \hat{A}_{4'1'} = L_{4'1'4'}^{4'1'4'} \hat{A}_{41}^4 + L_{4'1'1'}^{4'4'4'} \hat{A}_{44}^4 = \alpha_C^3 \hat{U}_1 - \alpha_C (i\alpha_C v_C) (-i\alpha_C v_C) \hat{U}_1 = \alpha_C \hat{U}_1$$

$$\hat{U}_{4'} = \hat{A}_{1'4'} = L_{1'4'4'}^{4'1'1'} \hat{A}_{41}^4 + L_{1'4'1'}^{4'4'1'} \hat{A}_{44}^4 = (i\alpha_C v_C)^2 (-i\alpha_C v_C) \hat{U}_1 - \alpha_C^2 (i\alpha_C v_C) \hat{U}_1 = -i\alpha_C v_C \hat{U}_1$$

#14.....

$$'L_{1'} = 'L_{4'1'} = L_s^4 L_{1'4'}^s = L_1^4 L_{1'4'}^1 + L_4^4 L_{1'4'}^4 = -i\alpha_C v_C \alpha_{C|4'} + \alpha_C (i\alpha_C v_C)_{|4'} = i\alpha_C^2 v_{C|4'}$$

$$= i(\alpha_R^2 v_{R|4'} - \alpha_1^2 v_{|4'}) = -i\alpha_C v_C i\alpha_R^2 v_{R|1} = \alpha_C v_C \alpha_R^2 \left(-a_R \frac{1}{R_g} \right) = -\alpha_C v_C \alpha_R \frac{1}{R_g} = -G_{1'}$$

$$'L_{4'} = 'L_{1'4'} = L_s^1 L_{4'|1'}^s = L_1^1 L_{4'|1'}^1 + L_4^1 L_{4'|1'}^4 = \alpha_C (-i\alpha_C v_C)_{|1'} + i\alpha_C v_C \alpha_{C|1'} = -i\alpha_C^2 v_{C|1'}$$

$$= -i(\alpha_R^2 v_{R|1'} - \alpha_1^2 v_{|1'}) = -i \left(-\alpha_C \alpha_R^2 a_R \frac{1}{R_g} - \alpha_1^2 a_1 v_1 \frac{1}{r} \right) = i\alpha_C \alpha_R \frac{1}{R_g} + i\alpha_1 v_1 \frac{1}{r} = -G_{4'} + I_{4'}$$

#15.....

$$g_{1'} = \hat{U}_{1'} - G_{1'} = \alpha_C \alpha_R v_R \frac{1}{R_g} - \alpha_C v_C \alpha_R \frac{1}{R_g} = \alpha_C \alpha_R (v_R - v_C) \frac{1}{R_g} = \alpha_1 v_1 \frac{1}{R_g}$$

$$'U_{1'} = p \alpha_1 v_1 \frac{1}{R_g}$$

$$'U_{4'} = \hat{U}_{4'} - G_{4'} + I_{4'} = -i\alpha_C v_C \alpha_R v_R \frac{1}{R_g} + i\alpha_C \alpha_R \frac{1}{R_g} + i\alpha_1 v_1 \frac{1}{r}$$

$$= i\alpha_C \alpha_R (1 - v_C v_R) \frac{1}{R_g} + i\alpha_C \alpha_R (v_R - v_C) \frac{1}{r}$$

$$= i\alpha_1 \frac{1}{R_g} + i\alpha_1 v_1 \frac{1}{r} = i\alpha_1 (v_1 - v_R) \frac{1}{r} = -i\alpha_C v_C \frac{1}{\alpha_R r} = -i\alpha_C v_C a_R \frac{1}{r}$$

#16.....

$$T_{||n'}^{4'n'} = T_{|n'}^{4'n'} + 'A_{n'r'}^{4'} T_{r'n'}^{4'} + 'A_{n'} T_{4'n'}^{4'}$$

$$= T_{|4'}^{4'4'} + 'A_{1'1'}^{4'} T_{1'1'}^{4'} + 'A_{2'2'}^{4'} T_{2'2'}^{4'} + 'A_{3'3'}^{4'} T_{3'3'}^{4'} + 'A_{4'} T_{4'4'}^{4'}$$

$$= \mu_{0|4'} + p('U_{4'} + B_{4'} + C_{4'}) + \mu_0 ('U_{4'} + B_{4'} + C_{4'}) = 0$$

$$\mu_{0|4'} = -3(p + \mu_0) 'U_{4'}$$

$$\kappa \mu_{0|4'} = \left(\frac{3}{R_g^2} \right)_{|4'} = -2 \cdot 3 \frac{1}{R_g^3} R_{4'} = -2 \kappa \mu_0 F_{4'}, \quad \mu_{0|4'} = -2 \mu_0 F_{4'}$$

$$2 \mu_0 F_{4'} = 3(p + \mu_0) 'U_{4'} = 2(1 - p) \mu_0 'U_{4'}$$

$$F_{4'} = (1 - p) 'U_{4'} = -i\alpha_C v_C (1 - p) \frac{a_R}{r}, \quad F_{1'} = 0, \quad F_m = \{-i\alpha_C v_C, 0, 0, \alpha_C\} (-i\alpha_C v_C) (1 - p) \frac{a_R}{r}$$

#17.....

$$\begin{aligned}
 \mathbf{T}'_{n'} \parallel_{n'} &= \mathbf{T}'_{n'} \parallel_{n'} + {}^1\mathbf{A}_{n'r'} \mathbf{T}'_{r'n'} + {}^1\mathbf{A}_{n'} \mathbf{T}'_{n'} \\
 &= \mathbf{T}'_{1'1'} + {}^1\mathbf{A}_{2'2'} \mathbf{T}'_{2'2'} + {}^1\mathbf{A}_{3'3'} \mathbf{T}'_{3'3'} + {}^1\mathbf{A}_{4'4'} \mathbf{T}'_{4'4'} + {}^1\mathbf{A}_{1'} \mathbf{T}'_{1'1'} \\
 &= -\rho_{1'} + \rho(\mathbf{B}_{1'} + \mathbf{C}_{1'}) - \mu_0 {}^1\mathbf{U}_{1'} - \rho(\mathbf{B}_{1'} + \mathbf{C}_{1'} + {}^1\mathbf{U}_{1'}) \\
 &= -\rho_{1'} - (\rho + \mu_0) {}^1\mathbf{U}_{1'} = 0
 \end{aligned}$$

$$\rho_{1'} = -(\rho + \mu_0) {}^1\mathbf{U}_{1'}$$

$$\kappa \rho_{1'} = \left[-\frac{1}{\mathcal{R}_g^2} (1 + 2\rho) \right]_{1'} = -\frac{2}{\mathcal{R}_g^2} \rho_{1'}$$

$$\kappa(\rho + \mu_0) {}^1\mathbf{U}_{1'} = \frac{2}{\mathcal{R}_g^2} \rho_{1'}$$

$$\rho_{1'} = (1 - \rho) {}^1\mathbf{U}_{1'}$$

#18.....

$$\mathbf{v}_{R|1} = \left(-\frac{\mathbf{r}}{\mathcal{R}_g} \right)_{1'} = -\frac{1}{\mathcal{R}_g} \mathbf{r}_{1'} + \frac{\mathbf{r}}{\mathcal{R}_g^2} \mathcal{R}_{g|1} = -\frac{\mathbf{a}_R}{\mathcal{R}_g} - \mathbf{v}_R \mathcal{F}_{1'}, \quad \mathbf{v}_{R|4} = \left(-\frac{\mathbf{r}}{\mathcal{R}_g} \right)_{4'} = -\frac{1}{\mathcal{R}_g} \mathbf{r}_{4'} + \frac{\mathbf{r}}{\mathcal{R}_g^2} \mathcal{R}_{g|4} = -\mathbf{v}_R \mathcal{F}_{4'}$$

$$\mathbf{v}_{R|m} = \{1, 0, 0, 0\} \left(-\frac{\mathbf{a}_R}{\mathcal{R}_g} \right) - \mathbf{v}_R \mathcal{F}_m$$

$$\frac{1}{\alpha_R} \alpha_{R|1} = \alpha_R^2 \mathbf{v}_R \mathbf{v}_{R|1} = \alpha_R^2 \mathbf{v}_R \left(-\frac{\mathbf{a}_R}{\mathcal{R}_g} - \mathbf{v}_R \mathcal{F}_{1'} \right) = -\alpha_R \mathbf{v}_R \frac{1}{\mathcal{R}_g} - \alpha_R^2 \mathbf{v}_R^2 \mathcal{F}_{1'}$$

$$\frac{1}{\alpha_R} \alpha_{R|4} = \alpha_R^2 \mathbf{v}_R \mathbf{v}_{R|4} = \alpha_R^2 \mathbf{v}_R (-\mathbf{v}_R \mathcal{F}_{4'}) = -\alpha_R^2 \mathbf{v}_R^2 \mathcal{F}_{4'}$$

$$\frac{1}{\alpha_R} \alpha_{R|m} = -\hat{\mathbf{U}}_m - \alpha_R^2 \mathbf{v}_R^2 \mathcal{F}_m, \quad \hat{\mathbf{U}}_m = \{1, 0, 0, 0\} \alpha_R \mathbf{v}_R \frac{1}{\mathcal{R}_g}$$

$$\frac{1}{\alpha_R} \alpha_{R|m'} = -\hat{\mathbf{U}}_{m'} - \alpha_R^2 \mathbf{v}_R^2 \mathcal{F}_{m'}, \quad \hat{\mathbf{U}}_{m'} = \{\alpha_C, 0, 0, -i\alpha_C \mathbf{v}_C\} \alpha_R \mathbf{v}_R \frac{1}{\mathcal{R}_g}, \quad \mathcal{F}_{1'} = 0$$

$$\frac{1}{\alpha_R} \alpha_{R|1'} = -\hat{\mathbf{U}}_{1'}$$

$$\begin{aligned}
 \frac{1}{\alpha_R} \alpha_{R|4'} &= -\hat{\mathbf{U}}_{4'} - \alpha_R^2 \mathbf{v}_R^2 \mathcal{F}_{4'} = i\alpha_C \mathbf{v}_C \alpha_R \mathbf{v}_R \frac{1}{\mathcal{R}_g} - \alpha_R^2 \mathbf{v}_R^2 (-i\alpha_C \mathbf{v}_C) (1 - \rho) \frac{\alpha_R}{r} \\
 &= i\alpha_C \mathbf{v}_C \alpha_R \mathbf{v}_R \frac{1}{\mathcal{R}_g} - i\alpha_C \mathbf{v}_C (1 - \rho) \alpha_R \mathbf{v}_R \frac{1}{\mathcal{R}_g} = i\alpha_C \mathbf{v}_C \rho \alpha_R \mathbf{v}_R \frac{1}{\mathcal{R}_g} = -\rho \hat{\mathbf{U}}_{4'}
 \end{aligned}$$

$$\frac{1}{\alpha_R} \alpha_{R|m'} = \{-\hat{\mathbf{U}}_{1'}, 0, 0, -\rho \hat{\mathbf{U}}_{4'}\}$$

$$\begin{aligned}
\alpha_C^2 v_{C1'} &= \alpha_R^2 v_{R1'} - \alpha_I^2 v_{I1'} = \alpha_R^2 \left(-\alpha_C a_R \frac{1}{R_g} \right) - \alpha_I^2 a_I v_I \frac{1}{r} \\
&= -\alpha_C \alpha_R \frac{1}{R_g} - \alpha_I v_I \frac{1}{r} = -\alpha_C \alpha_R \frac{1}{R_g} - \alpha_C \alpha_R (v_R - v_C) \frac{1}{r} = \alpha_C v_C \alpha_R \frac{1}{r} \\
\alpha_C^2 v_{C4'} &= \alpha_R^2 v_{R4'} - \alpha_I^2 v_{I4'} = \alpha_R^2 \left(i \alpha_C v_C a_R \frac{1}{R_g} - v_R F_{4'} \right) = i \alpha_C v_C \alpha_R \frac{1}{R_g} - \alpha_R^2 v_R \left[-i \alpha_C v_C (1-p) \frac{a_R}{r} \right] \\
&= i \alpha_C v_C \alpha_R \frac{1}{R_g} - i \alpha_C v_C (1-p) \alpha_R \frac{1}{R_g} = i \alpha_C v_C \alpha_R p \frac{1}{R_g} \\
\frac{1}{\alpha_C} \alpha_{C1'} &= \alpha_C^2 v_C v_{C1'} = \alpha_C v_C^2 \alpha_R \frac{1}{r}, \quad \frac{1}{\alpha_C} \alpha_{C4'} = \alpha_C^2 v_C v_{C4'} = -i \alpha_C v_C^2 p \alpha_R \frac{1}{R_g}
\end{aligned}$$

$$\begin{aligned}
\alpha_C^2 v_{C1} &= \alpha_R^2 v_{R1} - \alpha_I^2 v_{I1} = \alpha_R^2 \left(-a_R \frac{1}{R_g} - v_R F_{1'} \right) - \alpha_I^2 \alpha_C a_I v_I \frac{1}{r} \\
&= -\alpha_R \frac{1}{R_g} - \alpha_R^2 v_R (-\alpha_C^2 v_C^2) (1-p) \frac{a_R}{r} - \alpha_C \alpha_I v_I \frac{1}{r} \\
&= -\alpha_R \frac{1}{R_g} - \alpha_C^2 v_C^2 (1-p) \alpha_R \frac{1}{R_g} - \alpha_C \alpha_I v_I \frac{1}{r} \\
&= -\alpha_C^2 \alpha_R \frac{1}{R_g} + \alpha_C^2 v_C^2 p \alpha_R \frac{1}{R_g} - \alpha_C^2 \alpha_R (v_R - v_C) \frac{1}{r} = \alpha_C^2 v_C \alpha_R \frac{1}{r} + \alpha_C^2 v_C^2 p \alpha_R \frac{1}{R_g} \\
v_{C1} &= v_C \alpha_R \frac{1}{r} + v_C^2 p \alpha_R \frac{1}{R_g}
\end{aligned}$$

$$\begin{aligned}
\alpha_C^2 v_{C4} &= \alpha_R^2 v_{R4} - \alpha_I^2 v_{I4} = -i \alpha_C^2 v_C (1-p) \alpha_R \frac{1}{R_g} - \alpha_I^2 i \alpha_C v_C a_I v_I \frac{1}{r} \\
&= -i \alpha_C^2 v_C \alpha_R \frac{1}{R_g} + i \alpha_C^2 v_C p \alpha_R \frac{1}{R_g} - i \alpha_C v_C \alpha_I v_I \frac{1}{r} \\
&= -i \alpha_C^2 v_C \alpha_R \frac{1}{R_g} + i \alpha_C^2 v_C p \alpha_R \frac{1}{R_g} - i \alpha_C v_C \alpha_C \alpha_R (v_R - v_C) \frac{1}{r} = i \alpha_C^2 v_C^2 \alpha_R \frac{1}{r} + i \alpha_C^2 v_C p \alpha_R \frac{1}{R_g} \\
v_{C4} &= i v_C^2 \alpha_R \frac{1}{r} + i v_C p \alpha_R \frac{1}{R_g}
\end{aligned}$$

#19.....

$$\begin{aligned}
 L_1 &= -i\alpha_C^2 v_{C|4} = -i(\alpha_R^2 v_{R|4} - \alpha_I^2 v_{I|4}) = -i\alpha_R^2 (-v_R F_4) - \alpha_C v_C \alpha_I v_I \frac{1}{r} \\
 &= i\alpha_R^2 v_R (1-p)(-i\alpha_C^2 v_C) \frac{a_R}{r} - l_1 = (1-p)\alpha_C^2 v_C \alpha_R v_R \frac{1}{r} - l_1 = -(1-p)\alpha_C^2 v_C \alpha_R \frac{1}{R_g} = -f_1 - l_1 \\
 L_4 &= i\alpha_C^2 v_{C|1} = i(\alpha_R^2 v_{R|1} - \alpha_I^2 v_{I|1}) = i \left[\alpha_R^2 \left(-\frac{a_R}{R_g} - v_R F_1 \right) - \alpha_I^2 a_I v_I \frac{1}{r} \right] \\
 &= -i\alpha_R \frac{1}{R_g} - i\alpha_R^2 v_R (1-p)(-i\alpha_C v_C)(-i\alpha_C v_C) \frac{a_R}{r} - \alpha_I v_I \frac{1}{r} \\
 &= G_4 - l_4 - (1-p)\alpha_C^2 v_C^2 \alpha_R \frac{1}{R_g} = G_4 - l_4 - f_4
 \end{aligned}$$

#20.....

$$\begin{aligned}
 {}^1R_{m'n'} &= L_{m'n}^{m'n} R_{mn} + {}^1L_{m'n'} \\
 {}^1L_{m'n'} &= {}^1L_{m'n'}^{s'} - {}^1L_{n'm'} - {}^1L_{r'm'}^{s'} {}^1L_{s'n'}^{r'} + {}^1L_{m'n'}^{s'} L_{s'} + 2A_{[m'r]}^{s'} {}^1L_{s'n'}^{r'} \\
 \text{I} + \text{II} &= [h_{m'}^{s'} {}^1L_{n'} - h_{m'n'} {}^1L^{s'}]_{|s'} - {}^1L_{n'm'} = {}^1L_{n'm'} - h_{m'n'} {}^1L_{|s'}^{s'} - {}^1L_{n'm'} = -h_{m'n'} {}^1L_{|s'}^{s'} \\
 \text{III} &= -[h_{r'}^{s'} {}^1L_{m'} - h_{r'm'} {}^1L^{s'}][h_{s'}^{r'} {}^1L_{n'} - h_{s'n'} {}^1L^{r'}] \\
 &= -[2{}^1L_{m'} {}^1L_{n'} - {}^1L_{m'} {}^1L_{n'} - {}^1L_{m'} {}^1L_{n'} + {}^1L_{m'} {}^1L_{n'}] = -{}^1L_{m'} {}^1L_{n'} \\
 \text{IV} &= [h_{m'}^{s'} {}^1L_{n'} - h_{m'n'} {}^1L^{s'}] {}^1L_{s'} = {}^1L_{m'} {}^1L_{n'} - h_{m'n'} {}^1L^{s'} {}^1L_{s'} \\
 \text{V} &= 2A_{[m'r]}^{s'} {}^1L_{s'n'}^{r'} = (U_{m'r'}^{s'} - U_{r'm'}^{s'}) (h_{s'}^{r'} {}^1L_{n'} - h_{s'n'} {}^1L^{r'}) \\
 &= -U_{m'} {}^1L_{n'} - U_{m'r'n'} {}^1L^{r'} + U_{r'm'n'} {}^1L^{r'} \\
 &= -U_{m'} {}^1L_{n'} - h_{m'n'} U_{r'} {}^1L^{r'} + h_{m'r'} U_{n'} {}^1L^{r'} + h_{r'n'} U_{m'} {}^1L^{r'} - h_{r'm'} U_{n'} {}^1L^{r'} \\
 &= -U_{m'} {}^1L_{n'} - h_{m'n'} U_{r'} {}^1L^{r'} + U_{n'} {}^1L_{m'} + U_{m'} {}^1L_{n'} - U_{n'} {}^1L_{m'} = -h_{m'n'} U_{r'} {}^1L^{r'} \\
 {}^1L_{m'n'} &= -h_{m'n'} [{}^1L_{|s'}^{s'} + U_s {}^1L^{s'} + {}^1L^{s'} {}^1L_{s'}] = -h_{m'n'} [{}^1L_{|s'}^{s'} + U_s {}^1L^{s'}] = 0 \\
 R_{m'n'} &= L_{m'n}^{m'n} R_{mn}, \quad {}^1L_{|s'}^{s'} + U_s {}^1L^{s'} = 0
 \end{aligned}$$

#21.....

$$\mathbf{G}_{1|1'} = \left(\alpha_C v_C \alpha_R \frac{1}{R_g} \right)_{|1'} = (\alpha_C v_C)_{|1'} \alpha_R \frac{1}{R_g} + \alpha_C v_C \alpha_{R|1'} \frac{1}{R_g} = \alpha_C^3 v_{C|1'} \alpha_R \frac{1}{R_g} + \frac{1}{\alpha_C} \alpha_{R|1'} \mathbf{G}_{1'}$$

$$\frac{1}{\alpha_R} \alpha_{R|m'} = -\hat{U}_{m'} - \alpha_R^2 v_R^2 \mathcal{F}_{m'} \quad \mathcal{F}_{1'} = 0$$

$$\mathbf{G}_{1|1'} = i \alpha_C^2 v_{C|1'} \left(-i \alpha_C \alpha_R \frac{1}{R_g} \right) - \hat{U}_{1'} \mathbf{G}_{1'} = -\mathbf{G}_{4'} 'L_{4'} - \hat{U}_{1'} \mathbf{G}_{1'}$$

$$\mathbf{G}_{4'|4'} = \left(-i \alpha_C \alpha_R \frac{1}{R_g} \right)_{|4'} = -i (\alpha_{C|4'} \alpha_R + \alpha_C \alpha_{R|4'}) \frac{1}{R_g} + i \alpha_C \alpha_R \frac{1}{R_g^2} R_{g|4'}$$

$$= \left(\frac{1}{\alpha_C} \alpha_{C|4'} + \frac{1}{\alpha_R} \alpha_{R|4'} \right) \mathbf{G}_{4'} - \mathbf{G}_{4'} \mathcal{F}_{4'} = (\alpha_C^2 v_C v_{C|4'} - \hat{U}_{4'} - \alpha_R^2 v_R^2 \mathcal{F}_{4'}) \mathbf{G}_{4'} - \mathcal{F}_{4'} \mathbf{G}_{4'}$$

$$\mathbf{G}_{4'|4'} = -'L_{1'} \mathbf{G}_{1'} - \hat{U}_{4'} \mathbf{G}_{4'} - \alpha_R^2 \mathcal{F}_{4'} \mathbf{G}_{4'}$$

$$'L_{4'} = -\mathbf{G}_{4'} + l_{4'} = i \alpha_C \alpha_R \frac{1}{R_g} + i \alpha_1 v_1 \frac{1}{r} = -i \alpha_C \alpha_R v_R \frac{1}{r} + i \alpha_C \alpha_R (v_R - v_C) \frac{1}{r} = -i \alpha_C v_C \alpha_R \frac{1}{r}$$

$$\alpha_R^2 \mathcal{F}_{4'} = -(1-p) i \alpha_C v_C \alpha_R \frac{1}{r} = (1-p) 'L_{4'}$$

$$\mathbf{G}_{4'|4'} = -'L_{1'} \mathbf{G}_{1'} - \hat{U}_{4'} \mathbf{G}_{4'} - (1-p) 'L_{4'} \mathbf{G}_{4'}$$

$$\mathbf{G}^{s'}_{|s'} + 'L_{s'} \mathbf{G}^{s'} = -\hat{U}^{s'} \mathbf{G}_{s'} - (1-p) \mathbf{G}_{4'} 'L_{4'}, \quad \hat{U}^{s'} \mathbf{G}_{s'} = \hat{U}^s \mathbf{G}_s = 0$$

$$\mathbf{G}^{s'}_{|s'} + 'L_{s'} \mathbf{G}^{s'} = -(1-p) \mathbf{G}_{4'} 'L_{4'}$$

$$l_{1'} = 0, \quad l_{4'} = i \alpha_1 v_1 \frac{1}{r}$$

$$l_{4'|4'} = -i \alpha_1 v_1 \frac{1}{r^2} r_{|4'} = -l_{4'} 'U_{4'}$$

$$l^s_{|s'} = -l^{s'} 'U_s$$

$$f_{1'} = (1-p) \mathbf{G}_{1'}, \quad f_{4'} = 0, \quad p_{|1'} = (1-p) 'U_{1'}$$

$$f_{1|1'} = (1-p) \mathbf{G}_{1|1'} - \mathbf{G}_{1'} p_{|1'} = (1-p) (-'L_{4'} \mathbf{G}_{4'} - \hat{U}_{1'} \mathbf{G}_{1'}) - \mathbf{G}_{1'} (1-p) 'U_{1'}$$

$$f^s_{|s'} = -(1-p) ('L_{4'} \mathbf{G}_{4'} + \hat{U}_{1'} \mathbf{G}_{1'} + 'U_{1'} \mathbf{G}_{1'})$$

$$'L^s_{|s'} = -\mathbf{G}^{s'}_{|s'} + l^s_{|s'} + f^s_{|s'}$$

$$= 'L^s \mathbf{G}_s + (1-p) 'L_{4'} \mathbf{G}_{4'}$$

$$-l^{s'} 'U_s$$

$$-(1-p) ['L_{4'} \mathbf{G}_{4'} + \hat{U}_{1'} \mathbf{G}_{1'} + 'U_{1'} \mathbf{G}_{1'}]$$

$$\begin{aligned}
'L^s_{|s'} + 'U_s, 'L^s &= [-G^s + I^s + f^s]'U_s + 'L^s G_s - I^s 'U_s - (1-p)(\hat{U}_1 + 'U_1)G_1 \\
&= -G^s 'U_s + 'L^s G_s + (1-p)G_1 'U_1 - (1-p)G_1 'U_1 - (1-p)\hat{U}_1 G_1 \\
&= -G^s 'U_s - \hat{U}_1 G_1 + p\hat{U}_1 G_1 = -G_1 p\hat{U}_1 - G_4 'U_4 - \hat{U}_1 G_1 + p\hat{U}_1 G_1 = -G^s \hat{U}_s = -G^s \hat{U}_s = 0
\end{aligned}$$

$$'L^s_{|s'} + 'U_s, 'L^s = 0$$

#22.....

$$\begin{aligned}
'U_1 &= p g_1 = p \hat{U}_1 - G_1, \quad 'U_4 = g_4 + l_4 \\
g_{1|1'} &= \left(\alpha_1 v_1 \frac{1}{R_g} \right)_{|1'} = \alpha_1^3 v_{|1'} \frac{1}{R_g} = \alpha_1^2 v_1 \frac{1}{r} \frac{1}{R_g} = -i \alpha_1 v_1 \frac{1}{r} \cdot i \alpha_1 \frac{1}{R_g}
\end{aligned}$$

$$g_{1|1'} = -g_4 l_4$$

$$g_{4|4'} = \left(i \alpha_1 \frac{1}{R_g} \right)_{|4'} = -i \alpha_1 \frac{1}{R_g^2} R_{g4'} = -g_4 F_{4'}, \quad \alpha_{|4'} = 0$$

$$g_{4|4'} = -g_4 F_{4'}, \quad F_{4'} = (1-p)'U_4$$

$$l_{4|4'} = -l_4 'U_4$$

$$'U_{1|1'} = p g_{1|1'} + g_1 p_{|1'} = -p g_4 l_4 + g_1 (1-p)'U_1$$

$$'U^s_{|s'} = -p g_4 l_4 + g_1 'U_1 - 'U_1 'U_1 - g_4 F_{4'} - l_4 'U_4$$

$$\begin{aligned}
'U^s_{|s'} + 'U^s 'U_s &= 'U_4 'U_4 - l_4 'U_4 + g_1 'U_1 - p g_4 l_4 - g_4 (1-p)'U_4 \\
&= g_4 'U_4 + l_4 'U_4 - l_4 'U_4 + g_1 'U_1 - p g_4 l_4 - g_4 'U_4 + p g_4 'U_4 \\
&= p g_1 g_1 + p g_4 g_4 = p g^s g_s = p g^s g_s = -p \frac{1}{R_g^2}
\end{aligned}$$

$$'U^s_{|s'} + 'U^s 'U_s = -p \frac{1}{R_g^2}$$

#23.....

$$\mathbf{B}_{1'|1'} = \left(\alpha_C \mathbf{a}_R \frac{1}{r} \right)_{|1'} = -\alpha_C \mathbf{a}_R \frac{1}{r^2} r_{|1'} + \mathbf{B}_{1'} \left(\frac{1}{\alpha_C} \alpha_{C|1'} + \frac{1}{\mathbf{a}_R} \mathbf{a}_{R|1'} \right)$$

$$\begin{aligned} \mathbf{B}_{1'|1'} + \mathbf{B}_{1'} \mathbf{B}_{1'} &= \mathbf{B}_{1'} \left(\frac{1}{\alpha_C} \alpha_{C|1'} - \frac{1}{\alpha_R} \alpha_{R|1'} \right) = \mathbf{B}_{1'} (\alpha_C^2 v_C v_{C|1'} - \alpha_R^2 v_R v_{R|1'}) = \mathbf{B}_{1'} \left(\alpha_C v_C^2 \alpha_R \frac{1}{r} + \alpha_C \alpha_R v_R \frac{1}{R_g} \right) \\ &= \alpha_C^2 v_C^2 \frac{1}{r^2} + \alpha_C^2 v_R \frac{1}{r} \frac{1}{R_g} = \alpha_C^2 v_C^2 \frac{1}{r^2} - \alpha_C^2 \frac{1}{R_g^2} \end{aligned}$$

$$-{}^1\mathbf{U}_{1'1'} {}^4\mathbf{B}_{4'} = {}^1\mathbf{U}_{4'} \mathbf{B}_{4'} = \left(-i \alpha_C v_C \mathbf{a}_R \frac{1}{r} \right)^2 = -\alpha_C^2 v_C^2 \mathbf{a}_R^2 \frac{1}{r^2}$$

$$\mathbf{B}_{1' \parallel 1'} + \mathbf{B}_{1'} \mathbf{B}_{1'} = \mathbf{B}_{1' \parallel 1'} - {}^1\mathbf{U}_{1'1'} {}^4\mathbf{B}_{4'} + \mathbf{B}_{1'} \mathbf{B}_{1'}$$

$$= \alpha_C^2 v_C^2 (1 - \mathbf{a}_R^2) \frac{1}{r^2} - \alpha_C^2 \frac{1}{R_g^2} = \alpha_C^2 v_C^2 v_R^2 \frac{1}{r^2} - \alpha_C^2 \frac{1}{R_g^2} = (\alpha_C^2 v_C^2 - \alpha_C^2) \frac{1}{R_g^2} = -\frac{1}{R_g^2}$$

$$\mathbf{B}_{4'|1'} = \left(i \alpha_C v_C \mathbf{a}_R \frac{1}{r} \right)_{|1'} = -i \alpha_C \mathbf{a}_R \frac{1}{r^2} r_{|1'} - i \mathbf{a}_R \frac{1}{r} (\alpha_C v_C)_{|1'} + \mathbf{B}_{4'} \frac{1}{\mathbf{a}_R} \mathbf{a}_{R|1'}$$

$$\begin{aligned} \mathbf{B}_{4'|1'} + \mathbf{B}_{4'} \mathbf{B}_{1'} &= -i \mathbf{a}_R \frac{1}{r} \alpha_C^3 v_{C|1'} + \mathbf{B}_{4'} \hat{\mathbf{U}}_{1'} = -i \mathbf{a}_R \frac{1}{r} \alpha_C^2 v_C \alpha_R \frac{1}{r} + \mathbf{B}_{4'} \hat{\mathbf{U}}_{1'} \\ &= -i \alpha_C v_C \mathbf{a}_R \frac{1}{r} \cdot \alpha_C \alpha_R \frac{1}{r} + \mathbf{B}_{4'} \hat{\mathbf{U}}_{1'} = \mathbf{B}_{4'} \left(\alpha_C \alpha_R \frac{1}{r} + \alpha_C \alpha_R v_R \frac{1}{R_g} \right) \end{aligned}$$

$$= \mathbf{B}_{4'} \alpha_C \alpha_R (1 - v_R^2) \frac{1}{r} = \mathbf{B}_{4'} \alpha_C \mathbf{a}_R \frac{1}{r} = \hat{\mathbf{U}}_{4'} \mathbf{B}_{1'} = \hat{\mathbf{U}}_{14'} {}^1\mathbf{B}_{1'}$$

$$\mathbf{B}_{4' \parallel 1'} + \mathbf{B}_{4'} \mathbf{B}_{1'} = 0$$

$$\mathbf{B}_{1'|4'} = \left(\alpha_C \mathbf{a}_R \frac{1}{r} \right)_{|4'} = -\alpha_C \mathbf{a}_R \frac{1}{r^2} r_{|4'} + \mathbf{B}_{1'} \left(\frac{1}{\alpha_C} \alpha_{C|4'} + \frac{1}{\mathbf{a}_R} \mathbf{a}_{R|4'} \right)$$

$$\begin{aligned} \mathbf{B}_{1'|4'} + \mathbf{B}_{1'} \mathbf{B}_{4'} &= \mathbf{B}_{1'} \left(\frac{1}{\alpha_C} \alpha_{C|4'} - \frac{1}{\alpha_R} \alpha_{R|4'} \right) = \mathbf{B}_{1'} \left(i \alpha_C v_C^2 \rho \alpha_R \frac{1}{R_g} - i \alpha_C v_C \rho \alpha_R v_R \frac{1}{R_g} \right) \\ &= i \alpha_C \mathbf{B}_{1'} \alpha_C \alpha_R (v_C - v_R) \rho \frac{1}{R_g} = -i \alpha_C v_C \mathbf{B}_{1'} \rho \alpha_R v_R \frac{1}{R_g} = \mathbf{B}_{4'} {}^1\mathbf{U}_{1'} = {}^1\mathbf{U}_{4'1'} {}^4\mathbf{B}_{4'} \end{aligned}$$

$$\mathbf{B}_{1' \parallel 4'} + \mathbf{B}_{1'} \mathbf{B}_{4'} = 0$$

$$\mathbf{B}_{4'|4'} = \left(-i\alpha_C v_C a_R \frac{1}{r} \right)_{|4'} = -\mathbf{B}_{4'} \mathbf{B}_{4'} - i a_R \frac{1}{r} (\alpha_C v_C)_{|4'} + \mathbf{B}_{4'} \frac{1}{a_R} a_{R|4'}$$

$$\begin{aligned} \mathbf{B}_{4'|4'} + \mathbf{B}_{4'} \mathbf{B}_{4'} &= -i a_R \frac{1}{r} \alpha_C^3 v_{C|4'} + \mathbf{B}_{4'} \mathcal{P} \hat{\mathbf{U}}_{4'} = -\mathbf{B}_1 i \alpha_C \left(i \alpha_C v_C \mathcal{P} \alpha_R \frac{1}{R_g} \right) + \mathcal{P} \mathbf{B}_{4'} \hat{\mathbf{U}}_{4'} \\ &= \mathbf{B}_1 \alpha_C v_C \mathcal{P} \alpha_R \frac{1}{R_g} + \mathcal{P} \mathbf{B}_{4'} \hat{\mathbf{U}}_{4'} \end{aligned}$$

$$\begin{aligned} \mathbf{B}_{4'|4'} - {}^1\mathbf{U}_{4'4'} {}^1\mathbf{B}_1 + \mathbf{B}_{4'} \mathbf{B}_{4'} &= \mathbf{B}_1 \left({}^1\mathbf{U}_1 + \alpha_C v_C \alpha_R \mathcal{P} \frac{1}{R_g} \right) + \mathcal{P} \mathbf{B}_{4'} \hat{\mathbf{U}}_{4'} \\ &= \mathbf{B}_1 (\alpha_1 v_1 + \alpha_C v_C \alpha_R) \mathcal{P} \frac{1}{R_g} + \mathcal{P} \mathbf{B}_{4'} \hat{\mathbf{U}}_{4'} \end{aligned}$$

$$= \mathbf{B}_1 \alpha_C \alpha_R (v_R - v_C + v_C) \mathcal{P} \frac{1}{R_g} + \mathcal{P} \mathbf{B}_{4'} \hat{\mathbf{U}}_{4'} = \mathbf{B}_1 \alpha_C \alpha_R v_R \mathcal{P} \frac{1}{R_g} + \mathcal{P} \mathbf{B}_{4'} \hat{\mathbf{U}}_{4'}$$

$$= \mathcal{P} \mathbf{B}_1 \hat{\mathbf{U}}_1 + \mathcal{P} \mathbf{B}_{4'} \hat{\mathbf{U}}_{4'} = \mathcal{P} \mathbf{B}_1 \hat{\mathbf{U}}_1 = \mathcal{P} a_R \frac{1}{r} \alpha_R v_R \frac{1}{R_g} = -\mathcal{P} \frac{1}{R_g^2}$$

$$\mathbf{B}_{m' \| n'} + \mathbf{B}_m \mathbf{B}_{n'} = - \begin{pmatrix} 1 & & \\ & 0 & \\ & & 0 \end{pmatrix} \frac{1}{R_g^2}, \quad \mathbf{B}_{m' \| n'} = \mathbf{B}_{m' | n'} - {}^1\mathbf{U}_{n'm'} {}^s \mathbf{B}_{s'}, \quad \mathbf{B}_{s' \| s'} + \mathbf{B}^s \mathbf{B}_{s'} = -(1 + \mathcal{P}) \frac{1}{R_g^2}$$

#24.....

$$\mathbf{C}_{2' \| 2'} + \mathbf{C}_2 \mathbf{C}_{2'} = -\frac{1}{R_g^2}, \quad \text{siehe #5}$$

$$\mathbf{C}_{m' \| n'} = \mathbf{C}_{m' | n'} - {}^1\mathbf{U}_{n'm'} {}^s \mathbf{C}_{s'} - \mathbf{B}_{n'm'} {}^s \mathbf{C}_{s'}$$

$$\mathbf{C}_{s' \| s'} + \mathbf{C}^s \mathbf{C}_{s'} = -(2 + \mathcal{P}) \frac{1}{R_g^2}$$

$$\mathbf{C}_{m' \| n'} + \mathbf{C}_m \mathbf{C}_{n'} = - \begin{pmatrix} 1 & & \\ & 1 & \\ & & 0 \end{pmatrix} \frac{1}{R_g^2}$$

#25.....

$$\begin{aligned} -\frac{1}{2} \mathbf{R} &= \left[{}^1\mathbf{U}_{s' \| s'} + {}^1\mathbf{U}_{s'} {}^1\mathbf{U}_{s'} \right] + \left[\mathbf{B}_{s' \| s'} + \mathbf{B}^s \mathbf{B}_{s'} \right] + \left[\mathbf{C}_{s' \| s'} + \mathbf{C}^s \mathbf{C}_{s'} \right] \\ &= -\frac{\mathcal{P}}{R_g^2} - (1 + \mathcal{P}) \frac{1}{R_g^2} - (2 + \mathcal{P}) \frac{1}{R_g^2} = -(1 + \mathcal{P}) \frac{3}{R_g^2}, \quad \mathbf{R} = (1 + \mathcal{P}) \frac{6}{R_g^2} \end{aligned}$$

#26.....

$$G_{1'1'} = \frac{p}{R_g^2} + \frac{1}{R_g^2} + \frac{1}{R_g^2} - (1+p) \frac{3}{R_g^2} = -\frac{1}{R_g^2} (1+2p) = \kappa p, \quad T_{1'1'} = -p$$

$$G_{2'2'} = \frac{1}{R_g^2} + (1+p) \frac{1}{R_g^2} - (1+p) \frac{3}{R_g^2} = -\frac{1}{R_g^2} (1+2p) = \kappa p, \quad T_{2'2'} = -p$$

$$G_{3'3'} = (2+p) \frac{1}{R_g^2} - (1+p) \frac{3}{R_g^2} = -\frac{1}{R_g^2} (1+2p) = \kappa p, \quad T_{3'3'} = -p$$

$$G_{4'4'} = \frac{p}{R_g^2} + \frac{p}{R_g^2} + \frac{p}{R_g^2} - (1+p) \frac{3}{R_g^2} = -\frac{3}{R_g^2} = -\kappa \mu_0, \quad T_{4'4'} = \mu_0$$

$$G_{1'4'} = 0$$

#27.....

$$U_m = L_m^{m'} U_{m'} + L_m, \quad B_m = L_m^{m'} B_{m'}, \quad C_m = L_m^{m'} C_{m'}$$

$$\begin{aligned} U_1 &= L_1^{1'} U_{1'} + L_1^{4'} U_{4'} + L_1 = \alpha_C p g_1 - i \alpha_C v_C (-i \alpha_C v_C) a_R \frac{1}{r} + G_1 - f_1 - l_1 \\ &= \alpha_C p (\hat{U}_{1'} - G_{1'}) - \alpha_C^2 v_C^2 a_R \frac{1}{r} - \alpha_C v_C \alpha_1 v_1 \frac{1}{r} - \alpha_C (1-p) G_{1'} \\ &= \alpha_C p \hat{U}_{1'} - \alpha_C G_{1'} - \alpha_C^2 v_C^2 a_R \frac{1}{r} - \alpha_C v_C \cdot \alpha_C \alpha_R (v_R - v_C) \frac{1}{r} \\ &= \alpha_C^2 p \hat{U}_{1'} - \alpha_C^2 v_C \alpha_R \frac{1}{R_g} - \alpha_C^2 v_C^2 a_R \frac{1}{r} + \alpha_C^2 v_C \alpha_R \frac{1}{R_g} + \alpha_C^2 v_C^2 \alpha_R \frac{1}{r} \\ &= (1 + \alpha_C^2 v_C^2) p \hat{U}_{1'} + \alpha_C^2 v_C^2 \alpha_R (1 - a_R^2) \frac{1}{r} \\ &= p \hat{U}_{1'} + \alpha_C^2 v_C^2 \left(-p \alpha_R v_R^2 \frac{1}{r} + \alpha_R v_R^2 \frac{1}{r} \right) \\ &= p \hat{U}_{1'} + \alpha_R^2 v_R^2 (1-p) \alpha_C^2 v_C^2 a_R \frac{1}{r} \\ &= p \hat{U}_{1'} - \alpha_R^2 v_R^2 F_1 \end{aligned}$$

$$\begin{aligned}
U_4 &= L_4^{1'} U_1 + L_4^{4'} U_4 + L_4 = i\alpha_C v_C \mathcal{P}(\hat{U}_1 - G_1) + \alpha_C (-i\alpha_C v_C) a_R \frac{1}{r} + G_4 - f_4 - l_4 \\
&= i\alpha_C v_C \mathcal{P}(\hat{U}_1 - G_1) - i\alpha_C^2 v_C a_R \frac{1}{r} - i\alpha_R \frac{1}{R_g} - i\alpha_C \alpha_I v_I \frac{1}{r} - i\alpha_C^2 v_C^2 (1 - \mathcal{P}) \alpha_R \frac{1}{R_g} \\
&= i\alpha_C v_C \mathcal{P} \left(\alpha_C \alpha_R v_R \frac{1}{R_g} - \alpha_C v_C \alpha_R \frac{1}{R_g} \right) - i\alpha_C^2 v_C a_R \frac{1}{r} \\
&\quad - i\alpha_R \frac{1}{R_g} - \alpha_C^2 \alpha_R (v_R - v_C) \frac{1}{r} - i\alpha_C^2 v_C^2 \alpha_R \frac{1}{R_g} + \mathcal{P} i\alpha_C^2 v_C^2 \alpha_R \frac{1}{R_g} \\
&= i\alpha_C^2 v_C \mathcal{P} \alpha_R v_R \frac{1}{R_g} - i\alpha_C^2 v_C a_R \frac{1}{r} - (1 + \alpha_C^2 v_C^2) \alpha_R \frac{1}{R_g} + i\alpha_C^2 \alpha_R \frac{1}{R_g} + i\alpha_C^2 v_C \alpha_R \frac{1}{r} \\
&= i\alpha_C^2 v_C \mathcal{P} \alpha_R v_R \frac{1}{R_g} - i\alpha_C^2 v_C \alpha_R (1 - v_R^2) \frac{1}{r} + i\alpha_C^2 v_C \alpha_R \frac{1}{r} \\
&= i\alpha_C^2 v_C \mathcal{P} \alpha_R v_R \frac{1}{R_g} - i\alpha_C^2 v_C \alpha_R v_R \frac{1}{R_g} \\
&= -\alpha_R^2 v_R^2 (-i\alpha_C^2 v_C) (1 - \mathcal{P}) a_R \frac{1}{r} \\
&= -\alpha_R^2 v_R^2 \mathcal{F}_4
\end{aligned}$$

$$U_m = \mathcal{P} \hat{U}_m - \alpha_R^2 v_R^2 \mathcal{F}_m = -E_m - \alpha_R^2 v_R^2 \mathcal{F}_m$$

#28.....

$$'U^{s'}_{|s'} + 'U^{s'} U_{s'} = -\frac{\mathcal{P}}{R_g^2}, \quad 'U^{s'} = U^{s'} + 'L^{s'}$$

$$U^{s'}_{|s'} + 'L^{s'}_{|s'} + (U^{s'} + 'L^{s'}) U_{s'} = -\frac{\mathcal{P}}{R_g^2}$$

$$U^{s'}_{|s'} + U^{s'} U_{s'} + 'L^{s'}_{|s'} + 'U_{s'} 'L^{s'} = -\frac{\mathcal{P}}{R_g^2}$$

$$U^{s'}_{|s'} + 'U_{s'} U^{s'} = -\frac{\mathcal{P}}{R_g^2}$$

#29.....

$$'U^{s'}_{|s'} = (L^{s'} U^s)_{|s'} = 'U^s_{|s'} + 'U^s L^{s'}_{|s'} = U^s_{|s'} - L^s_{|s'} + 'U^s L_s$$

$$\begin{aligned}
'U^{s'}_{|s'} + 'U^{s'} U_{s'} &= U^s_{|s'} - L^s_{|s'} + 'U^s (U_s - L_s) + 'U^s L_s \\
&= U^s_{|s'} + U^s U_s - (L^s_{|s'} + U_s L^s) = U^s_{|s'} + U^s U_s
\end{aligned}$$

$$U^s_{|s'} + U^s U_s = 'U^{s'}_{|s'} + 'U^{s'} U_{s'} = -\frac{\mathcal{P}}{R_g^2}$$

#30.....

$$\mathbf{B}_{1|1} = \left(\frac{\mathbf{a}_R}{r} \right)_{|1} = -\mathbf{B}_1 \mathbf{B}_1 + \mathbf{B}_1 \frac{1}{\mathbf{a}_R} \mathbf{a}_{R|1}$$

$$\mathbf{B}_{1|1} + \mathbf{B}_1 \mathbf{B}_1 = -\mathbf{B}_1 \frac{1}{\alpha_R} \alpha_{R|1} = -\mathbf{B}_1 (-\hat{\mathbf{U}}_1 - \alpha_R^2 v_R^2 \mathcal{F}_1) = \frac{\mathbf{a}_R}{r} \alpha_R v_R \frac{1}{\mathcal{R}_g} - \alpha_R v_R \frac{1}{\mathcal{R}_g} \mathcal{F}_1 = -\frac{1}{\mathcal{R}_g^2} - \hat{\mathbf{U}}_1 \mathcal{F}_1$$

$$\mathbf{B}_{1|4} = -\mathbf{B}_1 \frac{1}{\alpha_R} \alpha_{R|4} = -\mathbf{B}_1 (-\alpha_R^2 v_R^2 \mathcal{F}_4) = -\hat{\mathbf{U}}_1 \mathcal{F}_4$$

$$\mathbf{B}_{4|1} = 0, \quad \mathbf{B}_{4|4} = 0$$

$$\mathbf{B}_{4|1} - \mathbf{U}_{14}^{-1} \mathbf{B}_1 + \mathbf{B}_4 \mathbf{B}_1 = -\mathbf{U}_4 \mathbf{B}_1 = \mathbf{B}_1 \alpha_R^2 v_R^2 \mathcal{F}_4 = -\alpha_R v_R \frac{1}{\mathcal{R}_g} \mathcal{F}_4 = -\mathcal{F}_4 \hat{\mathbf{U}}_1$$

$$\mathbf{B}_{4|4} - \mathbf{U}_{44}^{-1} \mathbf{B}_1 + \mathbf{B}_4 \mathbf{B}_4 = \mathbf{U}_1 \mathbf{B}_1 = \mathbf{B}_1 \mathcal{P} \hat{\mathbf{U}}_1 - \mathbf{B}_1 \alpha_R^2 v_R^2 \mathcal{F}_1 = -\frac{\mathcal{P}}{\mathcal{R}_g^2} + \hat{\mathbf{U}}_1 \mathcal{F}_1$$

$$\mathbf{B}_{m||n} + \mathbf{B}_m \mathbf{B}_n = - \begin{pmatrix} 1 & & & \\ & 0 & & \\ & & 0 & \\ & & & \mathcal{P} \end{pmatrix} \frac{1}{\mathcal{R}_g^2} + \begin{pmatrix} -\hat{\mathbf{U}}_1 \mathcal{F}_1 & & -\hat{\mathbf{U}}_1 \mathcal{F}_4 \\ & 0 & & \\ & & 0 & \\ -\mathcal{F}_4 \hat{\mathbf{U}}_1 & & & \hat{\mathbf{U}}_1 \mathcal{F}_1 \end{pmatrix}, \quad \mathbf{B}_{\parallel s}^s + \mathbf{B}^s \mathbf{B}_s = -(1 + \mathcal{P}) \frac{1}{\mathcal{R}_g^2}$$

#31.....

$$2\hat{\mathbf{U}}_1 \mathcal{F}_1 = 2\alpha_R v_R \frac{1}{\mathcal{R}_g} \cdot (-\alpha_C^2 v_C^2) (1 - \mathcal{P}) \frac{\mathbf{a}_R}{r} = \alpha_C^2 v_C^2 (1 - \mathcal{P}) \frac{2}{\mathcal{R}_g} = \alpha_C^2 v_C^2 \kappa (\mathcal{P} + \mu_0)$$

$$2\hat{\mathbf{U}}_1 \mathcal{F}_4 = 2\alpha_R v_R \frac{1}{\mathcal{R}_g} \cdot (-i\alpha_C^2 v_C) (1 - \mathcal{P}) \frac{\mathbf{a}_R}{r} = i\alpha_C^2 v_C (1 - \mathcal{P}) \frac{2}{\mathcal{R}_g} = i\alpha_C^2 v_C \kappa (\mathcal{P} + \mu_0)$$

$$2\hat{\mathbf{U}}_1 \mathcal{F}_1 = \alpha_C^2 v_C^2 \kappa (\mathcal{P} + \mu_0), \quad 2\hat{\mathbf{U}}_1 \mathcal{F}_4 = i\alpha_C^2 v_C \kappa (\mathcal{P} + \mu_0)$$

#32.....

$$\begin{aligned} \mathbf{R}_{11} &= - \left[\mathbf{U}_{\parallel s}^s + \mathbf{U}^s \mathbf{U}_s \right] - \left[\mathbf{B}_{1||1} + \mathbf{B}_1 \mathbf{B}_1 \right] - \left[\mathbf{C}_{1||1} + \mathbf{C}_1 \mathbf{C}_1 \right] \\ &= \frac{\mathcal{P}}{\mathcal{R}_g^2} + \left[\frac{1}{\mathcal{R}_g^2} + \hat{\mathbf{U}}_1 \mathcal{F}_1 \right] + \left[\frac{1}{\mathcal{R}_g^2} + \hat{\mathbf{U}}_1 \mathcal{F}_1 \right] = (2 + \mathcal{P}) \frac{1}{\mathcal{R}_g^2} + 2\hat{\mathbf{U}}_1 \mathcal{F}_1 \end{aligned}$$

$$\mathbf{G}_{11} = (2 + \mathcal{P}) \frac{1}{\mathcal{R}_g^2} - (1 + \mathcal{P}) \frac{3}{\mathcal{R}_g^2} + 2\hat{\mathbf{U}}_1 \mathcal{F}_1 \quad \text{mit \#25}$$

$$= -\frac{1}{\mathcal{R}_g^2} (1 + 2\mathcal{P}) + 2\hat{\mathbf{U}}_1 \mathcal{F}_1$$

$$= \kappa \mathcal{P} + \alpha_C^2 v_C^2 \kappa (\mathcal{P} + \mu_0)$$

$$\mathbf{T}_{11} = -\mathcal{P} - \alpha_C^2 v_C^2 (\mathcal{P} + \mu_0)$$

$$R = (1 + \rho) \frac{6}{R_g^2} \quad \#25$$

$$\begin{aligned} R_{22} &= - \left[\mathbf{B}_{2||s}^s + \mathbf{B}^s \mathbf{B}_s \right] - \left[\mathbf{C}_{2||2} + \mathbf{C}_2 \mathbf{C}_2 \right] \\ &= (1 + \rho) \frac{1}{R_g^2} + \frac{1}{R_g^2} = (2 + \rho) \frac{1}{R_g^2} \end{aligned}$$

$$G_{22} = (2 + \rho) \frac{1}{R_g^2} - (1 + \rho) \frac{3}{R_g^2} = -\frac{1}{R_g^2} (1 + 2\rho) = \kappa \rho$$

$$T_{22} = -\rho$$

$$R_{33} = - \left[\mathbf{C}_{3||s}^s + \mathbf{C}^s \mathbf{C}_s \right] = (2 + \rho) \frac{1}{R_g^2}$$

$$T_{33} = -\rho$$

$$\begin{aligned} R_{44} &= - \left[\mathbf{U}_{1||s}^s + \mathbf{U}^s \mathbf{U}_s \right] - \left[\mathbf{B}_{4||4} + \mathbf{B}_4 \mathbf{B}_4 \right] - \left[\mathbf{C}_{4||4} + \mathbf{C}_4 \mathbf{C}_4 \right] \\ &= \frac{\rho}{R_g^2} + 2 \left[\frac{\rho}{R_g^2} - \hat{U}_1 \mathcal{F}_1 \right] = 3\rho \frac{1}{R_g^2} - 2\hat{U}_1 \mathcal{F}_1 \end{aligned}$$

$$\begin{aligned} G_{44} &= 3\rho \frac{1}{R_g^2} - (1 + \rho) \frac{3}{R_g^2} - 2\hat{U}_1 \mathcal{F}_1 = -\frac{3}{R_g^2} - 2\hat{U}_1 \mathcal{F}_1 \\ &= -\kappa \mu_0 - \alpha_C^2 v_C^2 \kappa (\rho + \mu_0) \end{aligned}$$

$$T_{44} = \mu_0 + \alpha_C^2 v_C^2 (\rho + \mu_0)$$

$$G_{41} = - \left[\mathbf{B}_{1||4} + \mathbf{B}_1 \mathbf{B}_4 \right] - \left[\mathbf{C}_{1||4} + \mathbf{C}_1 \mathbf{C}_4 \right] = 2\hat{U}_1 \mathcal{F}_4 = i\alpha_C^2 v_C \kappa (\rho + \mu_0)$$

$$T_{41} = -i\alpha_C^2 v_C (\rho + \mu_0)$$

$$T_{mn} = -\rho g_{mn} + (\rho + \mu_0) 'u_m 'u_n, \quad 'u_m = \{-i\alpha_C v_C, 0, 0, \alpha_C\}$$